Optimal Power And Subcarrier Allocation For OFDMA Systems

BY

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This thesis, written by **HAITHAM HASSAN SALEH,** under the direction of his thesis advisor Prof. Shokri Selim, and approved by his thesis committee, has been presented to and accepted by the Dean of graduate studies, in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE IN SYSTEMS ENGINEERING.

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Dedicated to

My Parents,

Brothers and Sister,

Your love and support are always the source of my strength.

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First and foremost, I thank ALLAH for all the blessings and wonderful opportunities He has bestowed upon me in my journey through life. It is only by His grace that I have had the ability and strength to overcome life's challenges. May peace and blessing be upon his prophet Mohammed (PBUH), his family and his companions.

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THESIS ABSTRACT

Name: HAITHAM HASSAN H. SALEH

Title: OPTIMAL POWER AND SUBCARRIER ALLOCATIONS FOR OFDMA SYSTEMS

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With a drastic increase in number of users accessing wireless services, there is a fair demand for having fast response and reliable transmission of data between users' devices and the source of sending and receiving data such as antennas. Multiuser orthogonal frequency division multiplexing is an advanced technology to achieving high performance of downlink capacities in wireless communication network systems. The capacity of the system (MU-OFDMA) is optimized when each subcarrier is allocated with the appropriate sufficient power. The problem is a mixed of integer and nonlinear programs whose a nonlinear program solution is a straight forward. However, the problem becomes more challenging when the subcarriers are assigned to users and some users may receive disproportional powers. Therefore, fairness constraint is included. The resulting problem is more difficult especially if the number of subcarriers and users are large. In addition, we exploited the properties of the nonlinear problem and reduced it to a line search problem that can be solved in a very little time. The fairness conditions may result in an infeasible problem if the total power available for allocation is not sufficiently large. We provide the condition for having feasible problem. Therefore, we propose a new formulation where we relax the fairness conditions using a controlled tolerance. Finally, one of the evolutionary heuristic methods called Simulated Annealing is introduced to tackle the integer problem. Solved examples are presented to show the output and performance of both the designed algorithm and the heuristic method.

Keywords: integer nonlinear program, OFDMA, simulated annealing, fairness condition.

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خلاصـــــــة الأطــروحة

الاسم: هـيـثـم حـسـن هــارون صــالح

العنوان: الحل الأمثل لتوزيع القنوات والطاقة في نطاق أنظمة الترددات العامودية المتعددة

الدرجة: ماجستير في العلوم

التخصص: هندسة النظم

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مع الزيادة الكبيرة في عدد مستخدمي الخدمات اللاسلكية ، أصبحت طريقة وسرعة نقل البيانات بين أجهزة المستخدمين ومصدر إرسال وإستقبال هذه البيانات مثل الهوائيات مطلبا هاما. نطاق أنظمة الترددات العامودية اللاسلكية هي تقنية واعده في العصر الحالي لتحقيق قدرات إستعابية عالية في نقل بيانات الأنظمة الخلوية واللاسلكية.

الحل الأمثل لهذه الأنظمة عندما يتم تخصيص وتوزيع القنوات للمستخدمين بالطاقة المناسبة لضمان نقل البيانات بشكل سلس وسهل. المعادلات والبرامج التي تمثل هذه الأنظمة عبارة عن نوعين من المعادلات و البرامج. النوع الأول يسمى البرنامج اللاخطي ويحتوي على معادلات لاخطية اما البرنامج الثاني يسمى البرنامج العددي الصحيح ويحتوي على متغيرات عددية إما صفرا او واحدا. الحل الأمثل لبرنامج المعادلات اللاخطية واضح وسهل ومع ذلك فإن هذه الأنظمة تصبح أكثر صعوبة عندما يتم تعيين بعض القنوات للمستخدمين بطاقة معينة تؤثر على طاقة القنوات المخصصة للمستخدمين الآخرين في إرسال وإستقبال البيانات. لذلك تم وضع شرط يسمى الشرط الإنصافي في استقبال وارسال البيانات عن طريق هذه القنوات. الأنظمة الناتجة مع هذا الشرط هي أكثر صعوبة خاصة إذا كان عدد القنوات والمستخدمين كبيرة جدا. لذلك تتعامل هذه الأطروحة في استغلال خصائص الأنظمة اللاخطية وحل هذه الأنظمة بإستخدام البحث الخطي في متغير واحد والتي يمكن حلها في وقت قصير جدا. وقد يؤدي شرط الإنصاف الى عدم وجود حل أمثل لهذه الأنظمة بسبب عدم وجود الطاقة الكافية لتزويد هذه القنوات لإستيفاء هذا الشرط. في هذه الأطروحه أيضا تم تقديم حل لمثل هذه الحالات بإعاده صياغه وبناء هذه البرامج والمعادلات. وأخيرا تم تقديم احد الأساليب العصرية في حل هذه الأنظمة يدعى بخوارزمية محاكاة تبريد جزيئات المواد وختمت الأطروحة بعرض بعض الأمثلة وتقديم توصيات و مقترحات للبحوث المستقبلية في هذا المجال.

درجة الماجستير في العلوم

جامعة الملك فهد للبترول و المعادن

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Chapter 1

INTRODUCTION

1.1 INTRODUCTION

Wireless communications for a long time ago has been associated with cellular telephony networks. In 1970s, the cellular network was introduced and enabled the wireless communications using a limited frequency for mobile users within a specific area [1].

Mobile communication is one of the most creative and valuable innovations, evolving the need of wireless connectivity in all kinds of environmental applications in the twentieth century. Future wireless communications are expected to be flexible and reliable enough for providing the digital data transferring services with satisfying the needs of different applications. Nowadays [2], many applications use wireless communication for transmitting digital data over a radio wave such as cellular phones, internet networks at homes, and private networks at work … etc. There are many types of transferring data wirelessly such as follows single carrier system, Wideband Code Division Multiple Access (WCDMA) network system, Orthogonal Frequency Division Multiplexing

(OFDM) wireless network system, Orthogonal Frequency Division Multiple Access (OFDMA) wireless network system, and Universal Mobile Telecommunications Systems (UMTS). One of the important factors that affect the wireless communication is the rates of transferring the data over the radio wave.

In particular, OFDMA is considered by most of the mentioned types of wireless communication systems for the future as a promising solution to meet the QoS requirements. OFDMA has been introduced as a multiple access scheme for achieving high data rates and it is one of the important candidates for the next generation of communication systems due to many advantages such as its capability for exploiting the frequency diversity in a multiple user (MU) environment. Basically, in OFDMA, the frequency band is divided into a large number of small narrow-bands called subcarriers, simultaneously with a certain amount of power, to users in order to transmit information [3], [4].

The foremost objective for the network operators, for communication network systems, is to provide fair services with Quality of Service (QoS) requirements for each subscriber. Identifying the performance limits of power and subcarrier allocation with the objective of satisfying all users QoS requirements is therefore a challenging problem [5].

The network operators today seem to have selected that OFDM and OFDMA offer many advantages for transferring data wirelessly than other types, because of the standard process features and their capability of exploiting multi-path fading, it becomes more significant with the increase in the number of subcarriers, to improve the performance of the network systems. It has been shown that dynamic allocation of subcarriers to different users by utilizing the knowledge of subcarrier conditions can significantly improve the system performance [6].

In practice, to optimize the overall system performance, allocating the right amount of power to each subcarrier and allocating those subcarriers to K users can be formulated and modeled as a combinatorial optimization problem whose the objective function and the constraints can be determined based on the nature of the network conditions and specifications. The common objective is to optimize the total capacity of the system for all users in the communication network.

The numbers of aspects that cover OFDM and OFDMA [7, 8], of course, very huge and only two aspects of them, power allocation and subcarrier allocation, are discussed in this thesis.

In this inductor chapter 1, the problem is stated and well defined in section 1.2 followed by thesis organization in section 1.3.

1.2 PROBLEM STATEMENT

OFDM divides bandwidth into N subcarriers to be allocated, simultaneously with needed power P, to different K users. Consequently, selecting subcarriers to be allocated for each user and assigning the right amount of power for each subcarrier is a combinatorial optimization problem.

For wireless network operators, identifying the best allocation of subcarriers to users with the right amount of power and capacity for each subcarrier are the main bottlenecks for achieving good performance in the network system. In most wireless communication systems, different users need different rates of transferring data by allowing users to subscribe to different levels of services which they can choose from. The amount of allocated capacity for each user in the system is usually proportional to the subscribed service level and is considered as part of Quality of Service (QoS) requirements. A constraint has been introduced and known as the proportional rate constraint, also known as the fairness constraint to ensure that the level of subscribing service required to each user is met in the problem of power and subcarrier allocation for OFDMA systems.

The problem of subcarrier and power allocations for MU-OFDMA system consists of two types of mathematical programs, both maximizing the system throughput and satisfy the constraints of fairness and the total available power. The first type that deals with the power allocations can be formulated as a Nonlinear Programming model while the second type that deals with the subcarrier allocations can be formulated as an Integer

Programming model. The optimal solution for this problem, combined both integer and nonlinear programming in one model, is generally hard to obtain.

The objective of this thesis is to construct a general comprehensive framework for the problem of power and subcarrier allocations for OFDMA systems under fairness constraints. In this thesis, we tackle the Nonlinear Programming problem first to find the optimal distributed power for subcarriers and then we study the Integer Programming problem to find the optimal allocations of subcarriers to users using Simulated Annealing.

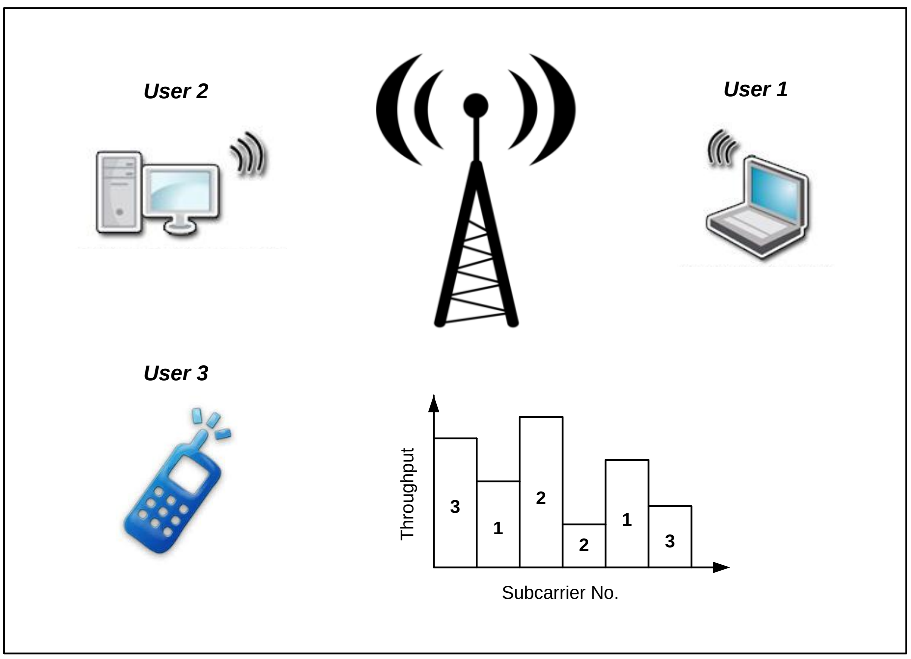

Figure 1-1 System Model

1.3 CONTENTS AND CONTRIBUTION OF THE THESIS

This thesis consists of five chapters, whose contents are briefly described in this section.

In this chapter, the introduction is addressed in order to provide some insights into the overall problem of maximizing the total capacity of the system, the nature of the problem, the components of the optimization problem, and the objectives of the thesis.

In Chapter 2, we present the general problem of the power and subcarrier allocation and we discuss the previous studies on relevant work. Firstly, we describe the general formulation of the power and subcarrier allocation that has been introduced in the literature. Further we discuss the previous studies that attempted to tackle and solve the problem.

In Chapter 3, we study the power allocation problem considering the fairness constraints. First, we apply the Karush-Kuhn-Tucker optimality conditions on the model. Next, we exploit the properties of the power allocation problem under fairness constraints using Karush-Kuhn-Tucker optimality conditions. In addition, we provide a new formulation where we relax fairness constraints using a controlled tolerance.

In Chapter 4, we present the power and subcarrier allocation with focus on subcarrier allocation. First, we introduce the power and subcarrier allocation model. Then, we introduce the Simulated Annealing heuristics. Later in this chapter we provide numerical examples and comparing the outcomes of the Simulated Annealing with the outcomes of the optimization software package LINGO 12.

In Chapter 5, we summarize the contributions of this thesis and present directions for future study.

Chapter 2

BACKGROUND AND RELATED WORK

2.1 BACKGROUND

In this chapter, the general mathematical program of the power and subcarrier allocations and a review of relevant previous work are presented.

Mathematical programming is a modeling technique for decision-making problems. In decision-making, identifying the possible decision to be taken leads to define the decision variables for the problem; which are the parameters whose values can be controlled and affect the system performance, one of the important properties of a decision variable. Then after defining the variables of the problem, this leads to define a value character that optimizes the objective of the problem, in other words, measures of effectiveness of the systems. The nature of the problem can be represented as a set of constraints that bounds the feasible region of the solution.

Through this thesis, we will refer to the form of mathematical program as the standard form as follows:

$$
\begin{aligned}
&min/max && f(x) \\
&subject\ to && g_i(x) \leq 0 \\
& && h_j(x) = 0
\end{aligned}
$$

where the objective function to be minimized or maximized is $f(x)$, $g_i(x), i = 1,2,3,\dots,m$ are the inequality constraints to the problem and $h_j(x)$, $j = 1,2,3,\dots,l$ are the equality constraints to the problem, and $m\ and\ l$ are the number of the constraints for the inequalities and the equalities, respectively.

Clearly, the decision variables that affect the system performance in the power and subcarrier allocation problem are the following:

$$\rho_{k,n} = \begin{cases} 1, & if\ subcarrier\ n\ is\ assigned\ to\ user\ k \\ 0, & otherwise \end{cases}$$

$$p_{k,n} = the\ amount\ of\ assigned\ power\ to\ subcarrier\ n\ for\ user\ k$$

The mathematical formulation of the power and subcarrier allocation has been introduced in different ways. Most of the studies in the literature address the same problem with relaxing in some constraints. In this thesis, we address the general mathematical program for the optimization problem including the fairness constraints.

The general mathematical formulation of the optimization problem of power and subcarrier allocations with typical constraints is described as follows:

1 - The objective function of the model

The objective function represents the total capacity of the system. Each user has a capacity which can be represented as

$$R_k = \sum_{n \in N(k)} \rho_{k,n} \log_2(1 + p_{k,n} H_{k,n}), \qquad k = 1,2,..,K$$

The function to be optimized is the sum of all capacities of all users in order to maximize the total capacity of the OFDMA system and it can be presented as follow

$$\text{Maximize} \sum_{k=1}^{K} \sum_{n \in N(k)} \rho_{k,n} \log_2(1 + p_{k,n} H_{k,n}) \qquad (2.1)$$

where

K	is the number of users
$N(k)$	is the set of sub-carriers assigned to user k, $\quad k = 1,2,\ldots,K$
P_{Total}	is the total available power
$p_{k,n}$	is the power allocated to user k on the subcarrier n
$H_{k,n}$	is the channel power gain for user k in subcarrier n
$\rho_{k,n}$	1 if subcarrier n is used by user k 0 otherwise

2 - The constraints of the model

The problem has four types of constraints that bound the feasible region of the solution. These are the following:

1. The total power constraint; the total power should not exceed the available power and it can be represented as follow

$$\sum_{k=1}^{K} \sum_{n \in N(k)} p_{k,n} \leq P_{Total} \qquad (2.2)$$

   where P_{Total} is the total available bandwidth and power. It is a straight forward to show that this constant is satisfied as equality at an optimal solution. In practice, the P_{Total} is known in advance.

2. The constraint that related to each subcarrier. It is assumed that each subcarrier can only be assigned to one user and it can be represented as follow

$$\sum_{k=1}^{K} \rho_{k,n} = 1, \quad n = 1,2, \dots N \qquad (2.3)$$

3. The set of fairness constraints; the proportional rate constraint that has to be met in order to satisfy the QoS requirements of each user and it can be represented as follow

$$R_1 : R_2 : \dots : R_k = \gamma_1 : \gamma_2 : \dots : \gamma_k \tag{2.4}$$

and it shows the proportional fairness among the users, where R_k is the total capacity of user k and $\gamma_i, i = 1,2, \dots K$ is a set of predetermined proportionality constants where

$$R_k = \sum\nolimits_{n \in N(k)} \rho_{k,n} \log_2(1 + p_{k,n} H_{k,n}) \tag{2.5}$$

In order to measure the proportional fairness among users, many researchers in the previous studies introduced an index called the fairness index. It can be defined as follows

$$\mathcal{F} = \frac{(\sum_{k=1}^{K} \gamma_k)^2}{K \sum_{k=1}^{K} {\gamma_k}^2} \tag{2.6}$$

$\mathcal{F}$ takes a maximum value of 1 in case of which all users would achieve the same rate of transferring data to satisfy the QoS requirements for each user.

4. The set of non-negativity constraint; the allocated power to each subcarrier must be nonnegative in order to satisfy the total power constraint.

$$p_{k,n} \geq 0, \quad n = 1,2, \dots, N \tag{2.7}$$

In other words, the standard form of the optimization problem of power and subcarrier allocation is shown in Figure 2-1.

$$\textit{Maximize} \qquad \sum_{k=1}^{K}\sum_{n\in N(k)} \rho_{k,n}\log_2\left(1+p_{k,n}H_{k,n}\right)$$

$$\textit{Subject to}$$

$$\sum_{k=1}^{K}\sum_{n\in N(k)} p_{k,n} \le P_{Total}$$

$$\sum_{k=1}^{K} \rho_{k,n} = 1$$

$$\frac{R_1}{\gamma_1} = \frac{R_2}{\gamma_2} = \frac{R_k}{\gamma_k}$$

$$p_{k,n} \ge 0$$

Figure 2-1 The power and subcarrier allocation optimization model

2.2 FOCUSED RELATED WORK

We divide the previous studies into two main categories. The researchers of the first category tackle the power allocation model while the researchers of the second category tackle the power and subcarrier allocation model simultaneously.

The first category, many researchers studied the problem of power allocation by assuming different assumption to simplify the problem. The description for this category as follow:

The study in [9], the authors used water-filling policy to distribute power over the subcarriers in order to maximize the data rate of a multiuser OFDM system. In addition, they found that found that the data rate of a multiuser OFDM system is maximized when each subcarrier is assigned to only one user with the best channel gain for that subcarrier under the assumption that a subcarrier can be shared among users.

The study in [10], the authors assumed in this study the perfect subcarrier state information at the transmitter and the receivers. They allocate the minimum required power for all subcarriers first and then tried to distribute the excess power in order to maximize the total system. In other words, they used a multi-level water-filling policy.

The study in [11], the authors introduced the fairness constraints in the power allocation model and tried to optimize the objective function and satisfy the fairness constraints using water-filling. Also, they proposed an algorithm by distributing first the power equally to all subcarriers and then allocate those subcarriers and then again redistribute the powers. Moreover, their algorithm achieved 95% of the optimal capacity in a two user system.

The study in [12], the authors proposed two types of algorithms. The first algorithm used quantized channel-to-noise ratio (CNR) to maximize the throughput while the second tried to let the receiver of data decides the constellation to be used in each subcarrier and sends back only the rate allocation vector to the transmitter. Moreover, both algorithms achieve significant efficiency gain have lower feedback requirements.

The study in [13], the authors proposed an iterative methods for power allocation with fairness constraints model. The algorithm distributed the power equal first and then allocated all subcarriers and then again tried to distribute the powers to subcarriers in order to satisfy the fairness constraints. The proposed algorithm provided a near optimal power for more than 98% of the minimum capacity requirements of each user.

The study in [14], the authors proposed an iterative algorithm that calculates the powers needed for each subcarrier for any given subcarrier allocation. Moreover, the proposed iterative algorithm achieved fairness constraints satisfaction.

The second category, many researchers studied the problem of power and subcarrier allocation by assuming different assumption to simplify the problem. The description for this category as follow:

The study in [15], the authors assumed the perfect channel information and derived a multiuser convex optimization problem in order to obtain an optimal allocation of subcarriers by proposing adaptive subcarrier allocation algorithm.

The study in [16], the authors assumed the knowledge of the instantaneous channel gains for all users and proposed an algorithm in order to optimize the total transmit power. The algorithm assigns each user a set of subcarriers and then determines the number of bits of the transmitted power for each subcarrier.

The study in [17], the authors proposed a non-iterative method that is made possible by relaxation of strict user rate proportionality constraints and this method waives the restriction of high subchannel SNR, has significantly lower complexity, and showed that in simulation study, the method yields higher user data rates.

The study in [18], the authors proposed a subcarrier allocation algorithm that gives fair allocation of capacity to users with different subcarrier and the characteristics of the traffic.

The study in [19], the authors proposed an algorithm in order to maximize the overall rate while achieving proportional fairness amongst users under a total power constraint. The proposed algorithm performed joint subcarrier and power allocation.

The study in [20], the authors proposed joint subcarrier and power allocation while allowing multiple users to share and use a single OFDM subcarrier. The algorithm tried to optimize the power allocation first and tried to allocate subcarriers.

The study in [21], the authors proposed an adaptive algorithm for power and subcarrier allocation in order to maximize the transmission data rate while satisfying total power constraint and a certain bit error rate (BER) requirement. The proposed algorithm performs two steps. The first step is the allocation of subcarriers and power alternately and the second step is the residual resource distribution.

The study in [22], the authors studied the sum throughput that maximize the total transmit power using water filling for power allocation model in order to satisfy the fairness constraints in different subcarriers parameters.

The study in [23], the authors studied the performance of low complexity adaptive resource allocation in the downlink of OFDMA systems with fixed or variable rate requirements putting fairness constraints into consideration. Moreover, the authors proposed an algorithm for maximizing the total throughput while maintaining rate proportionality among the users and showed that it is achieved the requirements.

In summary [11]-[24], the authors considered the proportional fairness constraint in the optimization problem. Many authors reduced the optimization problem to the subcarrier allocation problem by assuming that the power for each subcarrier is equal and proposed an iterative algorithm in order to satisfy the proportional rate constraint. Similarly, many authors discussed the subcarrier and power allocation by optimizing a user's power allocation, using waterfilling technique, after allocated a subcarrier for each user, in order to maintain the power allocated to other subcarriers to optimize the total capacity.

In addition, many authors proposed an algorithm and simplified the problem by assuming that the number of the allocated subcarriers for each user depends on the proportional ratio. Also, many authors utilized the same assumption depends on the gain variance for the allocated subcarriers to users and proposed an iterative algorithm in attempting to get the optimal solution for this type of problem.

Some authors in many studies proposed a method that assumes high and comparable subcarriers gains across the system bandwidth to find the power allocation for a given subcarriers distribution while other studies focused on satisfying the fairness constraint by assuming the power is distributed uniformly to the subcarriers and then proposed an iterative algorithm to find the power allocation for each user in order to satisfy the fairness constraints. Unmistakably, uniform distribution of powers among subcarriers yields to less quality of the solution.

The study in [24], attempted and tackled the problem differently using one of the evolutionary optimization techniques called genetic algorithm (GA).

This thesis tackles power allocations first by providing an algorithm to obtain the optimal power distribution for subcarriers and then using these as a sub routine to obtain the optimal subcarrier allocations for each user.

Chapter 3

THE POWER ALLOCATION MODEL

3.1 INTRODUCTION

In this chapter, we introduce the nonlinear model for the problem by assuming that sub-carrier allocations are known in advance for each user. Also, we exploit the properties of the problem using Karush-Kuhn-Tucker (KKT) optimality condition and reduced the optimization problem to a line search problem that can be solved in a very short time. However, the fairness constraints may result in an infeasible problem if the total power available for allocation is not sufficiently large. So, we provide a condition in order to make sure that the available power is sufficiently enough to obtain a feasible and optimal solution to the optimization problem.

In addition, we provide a condition for having feasible problem in case of not sufficient total power. Therefore, we propose a new formulation where we relax the fairness conditions using controlled tolerance.

3.2 MODEL I: POWER ALLOCATION PROBLEM WITH FAIRNESS CONSTRAINTS

This model's objective is to maximize the total capacity of the OFDMA system, i.e. the capacity for each user, without compromising the fairness condition in order to satisfy the QoS requirements for each user. This means that the problem is to find and obtain the power allocation to subcarriers in such a way the total capacity of all users is maximized. In other words, the weighted total power assigned to each user is the same for all users in the system.

So, we refer to these conditions as the "fairness" constraints and they are given as follows

$$\frac{\sum_{n=1}^{N(1)} \log_2(1 + p_{1,n}H_{1,n})}{\gamma_1} = \frac{\sum_{n=1}^{N(2)} \log_2(1 + p_{2,n}H_{2,n})}{\gamma_2} = \cdots = \frac{\sum_{n\in N(k)} \log_2(1 + p_{k,n}H_{k,n})}{\gamma_k} \tag{3.1}$$

Clearly, the decision variable that affects the system performance in the power allocation problem is

$$p_{k,n} = the\ amount\ of\ assigned\ power\ to\ subcarrier\ n\ for\ user\ k$$

and to simplify the description and the presentation of this problem without any loss of generality, we assume that the subcarriers assigned to a user are labeled in a descending order of the gains, i.e. $H_{k,n-1} \geq H_{k,n}$, $n \in N(k)$, $1 \leq k \leq K$.

The Nonlinear Program for the power allocation model is described as follows:

1 – The objective function for Model I

As mentioned earlier in this chapter, the function to be optimized is the total capacity of the systems. In other words, it means the objective of this model is to maximize the capacity of each user and the objective function can be represented as follow

$$\text{Maximize} \sum_{k=1}^{K} \sum_{n \in N(k)} \log_2(1 + p_{k,n} H_{k,n}) \qquad (3.2)$$

where

K	is the number of users
$N(k)$	is the set of sub-carriers assigned to user k, $\quad k = 1,2,\ldots,K$
P_{Total}	is the total available power
$p_{k,n}$	is the power allocated to user k on the subcarrier n
$H_{k,n}$	is the channel power gain for user k in subcarrier n

It is noticeable that the objective function of the model is a logarithmic function; that is a concave function.

2 – The constraints for Model I

The problem has four types of constraints that bound the feasible region of the solution. These are the following:

1. The total power constraint; the total power should not exceed available power.

$$\sum_{k=1}^{K}\sum_{n\in N(k)} p_{k,n} \leq P_{Total} \tag{3.3}$$

where P_{Total} is the total available bandwidth and power. It is a straight forward to show that this constant is satisfied as equality at an optimal solution. In practice, the P_{Total} is known in advance.

2. The set of fairness constraints; in order to simplify the (3.2), we modify the original representation of the constraints by equating each fairness constraint for each user to a variable Y to simplify the analysis of the KKT conditions and link all the fairness constraints to a common variable. In other words, variable Y has a value that insures each user received the same weighted amount of power distributed to subcarriers. The fairness constraints can be represented and modeled as follows where Y is a variable to be determined

$$\frac{1}{\gamma_k}\sum_{n\in N(k)} \log_2\left(1 + p_{k,n}H_{k,n}\right) = Y, \quad 1 \leq k \leq K \tag{3.4}$$

3. The set of non-negativity constraints; the allocated power to each subcarrier must be nonnegative in order to satisfy the total power constraint in the systems as we defined earlier and the non-negativity constraints can be represented and modeled as follow

$$p_{k,n} \geq 0, \quad n = 1,2, \dots, N \tag{3.5}$$

In other words, the complete and the standard form of the optimization problem of power allocation is shown in Figure 3-1.

$$\begin{array}{ll} \mathit{Maximize} & \sum_{k=1}^{K} \sum_{n \in N(k)} \log_2\left(1 + p_{k,n} H_{k,n}\right) \\ \mathit{Subject\ to} & \\ & \sum_{k=1}^{K} \sum_{n \in N(k)} p_{k,n} \leq P_{Total} \\ & \frac{1}{\gamma_{\mathrm{k}}} \sum_{n \in N(k)} \log_2\left(1 + p_{k,n} H_{k,n}\right) = Y \\ & p_{k,n} \geq 0 \end{array}$$

Figure 3-1 The power allocation optimization model

3.2.1 KKT OPTIMALITY CONDITIONS ANALYSIS FOR MODEL I

As mentioned earlier in this thesis, we use KKT conditions [26-28] to exploit the properties of the power allocation model. Applying the KKT conditions to the model, we get the following

$$-\frac{H_{k,n}}{1+p_{k,n}H_{k,n}}+\frac{\mu_k}{\gamma_k}\frac{H_{k,n}}{1+p_{k,n}H_{k,n}}-\lambda_{k,n}+\pi=0,\ n\in N(k),\ \ 1\le k\le K \qquad (3.6)$$

where

π is the dual variable corresponding to constraints (3.3)

μ_k are the dual variables corresponding to (3.4), $n \in N(k)$, $1 \le k \le K$

$\lambda_{k,n}$ are the dual variables corresponding to (3.5), $n \in N(k)$, $1 \le k \le K$.

In addition to that, the sum of the corresponding dual variables to (3.4) must be equal to zero and it can be written as follow

$$\sum_{k=1}^{K}\mu_k=0 \qquad (3.7)$$

Moreover, the complementary slackness of the power allocation model can be repressed and modeled as follow

$$\lambda_{k,n}p_{k,n}=0,\ \ \lambda_{k,n}\ge 0, \qquad n\in N(k),\ \ 1\le k\le K \qquad (3.8)$$

For simplicity, we will temporarily assume that $p_{k,n} > 0$, $n \in N(k), 1 \leq k \leq K$. Consequently, the dual variables $\lambda_{k,n} = 0$ for $n \in N(k), 1 \leq k \leq K$, and the (3.6) can be simplified as follows

$$\frac{H_{k,n}}{1 + p_{k,n}H_{k,n}} = \frac{\pi\gamma_k}{\gamma_k - \mu_k} \quad , \ n \in N(k), \ 1 \leq k \leq K \tag{3.9}$$

Subsequently, the (3.9) can be rearranged for each user and obtain the following result that shows the relationship between subcarriers for the same user, the result equations can be represented as follows

$$\frac{H_{k,n}}{1 + p_{k,n}h_{k,n}} = \frac{H_{k,l}}{1 + p_{k,l}h_{k,l}} \quad , \ n \in N(k), \ 1 \leq k, l \leq K \tag{3.10}$$

Consequently, for a given user, if the power allocated to some subcarrier is known; the powers allocated to all other subcarriers assigned to this user can be easily determined using the equations (3.10). Besides, The previous assumption that the subcarrier powers ($p_{k,n} > 0$) are strictly positive may be a practical necessity. This is because of assigning subcarriers to K users then allocating them zero power is most likely to be unacceptable for wireless network operators.

Using the equations (3.10) and making n_k^* is the label of the subcarrier with the least gain assigned to a user, we can write the following interpretation;

$$1 + p_{k,n} H_{k,n} = \left(1 + p_{k,n_k^*} H_{k,n_k^*}\right) \frac{H_{k,n}}{H_{k,n_k^*}}, \qquad n \in N(k), \qquad 1 \le k \le K \tag{3.11}$$

Subsequently, the equations (3.4) can be rewritten and modeled with some simplification as follow

$$\beta_k + \frac{|N(k)|}{\gamma_k} \log_2(1 + p_{k,n_k^*} H_{k,n_k^*}) = Y, \qquad 1 \le k \le K \tag{3.12}$$

where

$$\beta_k = \frac{1}{\gamma_k} \sum\nolimits_{n \in N(k)} \log_2 \left(\frac{H_{k,n}}{H_{k,n_k^*}} \right)$$

Subsequently, by modifying and rearranging the terms of the equations (3.12), we obtain the amount of power the must be allocated for the subcarrier that has least gain assigned to a user;

$$p_{k,n_k^*} = \frac{2^{(Y - \beta_k)\gamma_k / |N(k)|} - 1}{H_{k,n_k^*}}, \qquad 1 \le k \le K \tag{3.13}$$

Furthermore, we know that the fairness constraints for all users are equal Y. So, using the equations (3.12), for instance choosing any two users $k \; and \; t, \; 1 \le k, t \le K$,

$$\beta_k + \tfrac{|N(k)|}{\gamma_k} \log_2(1 + p_{k,n_k^*} H_{k,n_k^*}) = Y \qquad \text{and} \qquad \beta_t + \tfrac{|N(t)|}{\gamma_t} \log_2\left(1 + p_{t,n_t^*} H_{t,n_t^*}\right) = Y,$$

respectively. Consequently, we obtain the following observation

$$\beta_k + \frac{|N(k)|}{\gamma_k} \log_2\left(1 + p_{k,n_k^*} H_{k,n_k^*}\right) = \beta_t + \frac{|N(t)|}{\gamma_t} \log_2\left(1 + p_{t,n_t^*} H_{t,n_t^*}\right), \quad 1 \leq k, t \leq K \tag{3.14}$$

Hence, having some knowledge of the power allocated to a subcarrier of least gain of a user can be used to find and obtain the power of the least gain subcarrier for any *other* user. This observation along with that made after (3.10) implies that given the power allocation to any subcarrier of any user will result in the solution of the problem on hand.

In fact, before simplifying the (3.3), we notice that from (3.10) we can obtain the following result

$$p_{k,n} = p_{k,n_k^*} + \frac{1}{H_{k,n_k^*}} - \frac{1}{H_{k,n}} \ , \quad n \in N(k), 1 \leq k \leq K \tag{3.15}$$

which implies that $p_{k,n-1} \geq p_{k,n}$, $n \in N(k)$. In other words, having some knowledge of power for a least gain subcarrier for a user leads to find the allocated powers for other subcarriers.

Subsequently, we substitute the equation (3.13) and the equation (3.15) into the equation (3.3) in order to obtain the following result

$$\sum_{k=1}^{K}\sum_{n\in N(k)} p_{k,n} = \sum_{k=1}^{K} |N(k)| p_{k,n_k^*} + \sum_{k=1}^{K} \alpha_k$$
$$= \sum_{k=1}^{K} \frac{|N(k)|}{H_{k,n_k^*}}\left(2^{(Y-\beta_k)\gamma_k/|N(k)|} - 1\right) + \sum_{k=1}^{K} \alpha_k \qquad (3.16)$$
$$= P_{Total}, \qquad 1 \le k \le K$$

where

$$\alpha_k = \sum_{n\in N(k)} \left(\frac{1}{H_{k,n_k^*}} - \frac{1}{H_{k,n}}\right)$$

The (3.12) implies that $Y \geq \max_{1\leq k\leq K}\{\beta_k\}$. This lower bound on Y has some significance. The smaller the least subcarrier gain relative to other subcarrier of the user k, the higher is β_k. Similarly, the smaller γ_k, the higher is β_k. So, the lower bound signifies the subcarrier with most variation between the least gain and the rest of the gains with the smaller weight.

After that, we address the question, what is the least value of P_{Total} that insures each sub-carrier has strictly positive power and at the same time the fairness constraints are satisfied? From the equation (3.16), the least total power is that where Y is equal to its lower bound.

Suppose that $Y = \max_{1\leq k\leq K}\{\beta_k\} = \beta_t$ for some user t. From the equation (3.13), if the maximum of β_k is equal to β_t, then we can conclude that $P_{t,N(t)} = 0$. In other words, the least total power needed to have a feasible solution to the power allocation problem is given by the following observation

$$P_{least} = \sum_{k=1}^{K} \frac{|N(k)|}{h_{k,N(k)}} \left(2^{(\beta_t - \beta_k)\gamma_k / |N(k)|} - 1\right) + \sum_{k=1}^{K} \alpha_k \qquad (3.17)$$

If $P_{Total} < P_{least}$, the optimization problem is tend to be infeasible since the fairness constraints cannot be satisfied with positive power allocation for the subcarriers. Hence, we proposed an algorithm based on the finding results of applying KKT optimality conditions to the power allocation model and also propose a method for

3.2.2 THE PROPOSED ALGORITHM FOR MODEL I

Based on the finding of applying the KKT optimality conditions to the model, the proposed algorithm can be presented as follows:

Step 1: Compute the value of Y using (3.16), note that the term before the last in (3.16) is the sum of increasing functions; hence it is logarithmic function and a unique solution exists. One of many approaches to solve (3.16) is using a bisection search method. For the bisection search method, the range of Y where the solution of (3.16) exists is needed and this range is given by the following

$$\max_{1 \le k \le K} \{\beta_k\} \le Y \le \max_{1 \le k \le K} \left\{\log_2\left(1 + P_{Total} H_{1,n}\right) / \gamma_k\right\} \qquad (3.18)$$

in this step, we will get the value of Y which represents the proportional fairness for each user.

Step 2: Compute p_{k,n_k^*} for each user k, $1 \leq k \leq K$ using (3.13). In this step, we will obtain the allocated powers for a smallest gain subcarrier for each user.

Step 3: Compute $p_{k,n}$ for each user k, $1 \leq k \leq K$ using (3.15). In this step, we will obtain the allocated powers for all subcarriers.

3.2.3 POWER LEVELS FOR OFDMA SYSTEMS

In this section of the thesis, we consider the case where the powers assigned to subcarriers, $p_{k,n}$, are allowed to get value of zeros or in other words, receive no powers. If P_{Total} is smaller than P_{least} that has been introduced in equation (3.17), some subcarrier(s) will receive no power or get value of zero. Subsequently, as P_{Total} gets smaller and smaller, additional subcarriers belong to different users will have no power or get value of zero.

Moreover, the next step is to figure out the levels of P_{Total} that cause subcarrier powers to disappear. These levels are governed by the value of the maximum β. Subsequently, the calculation of β_k depends on the gain of the subcarrier which has the least gain assigned to the user. When these subcarriers receive no power, they are removed from the user's list of subcarriers and consequently a new value of β_k is calculated and a new value for maximum β is generated to be considered as a reference for the next step.

In order to compute the levels of the total power, we set and generalize the definition of β as follows;

Let $\beta_k^m = \frac{1}{\gamma_k}\sum_{n=1}^{m} \log_2\left(\frac{H_{k,n}}{H_{k,m}}\right),\ \ m \in N(k),\ 1 \leq k \leq K.$

The above definition is needed for accommodating the case where some low gain subcarriers have been removed from the user's subcarriers list. In similar manner, we set and generalize the definition of α_k as follows;

$$\alpha_k^m = \sum_{n=1}^{m}\left(\frac{1}{H_{k,m}} - \frac{1}{H_{k,n}}\right)$$

In order to find the total power levels, we implement the following steps;

Step 1: Compute β_k^m for each user k, and all m, where $m = 1,2, \ldots, N$.

Note: There are $\sum_{k=1}^{K} N(k) = N$ of these quantities.

In this step, we calculate the values of β for each user in order to identify the maximum value to be considered as a reference for the followed steps.

Step 2: Arrange these quantities in descending order. Suppose that the resulting sequence is $\beta_{(1)}, \beta_{(2)}, \ldots, \beta_{(t)}, \ldots, \beta_{(N)}$. The subscript ($t$) refers to some user k_t, and some subcarrier m_t.

Step 3: For each of these β values, identify which subcarrier will receive no power or get value of zero. Then, remove it from the list of subcarriers of the corresponding user *k*. *M(k)* be the remaining number of channels for user *k*.

Step 4: Compute for t =1,2,…, $\sum_{k=1}^{K} N(k)$

$$P_{(t)} = \sum_{k=1}^{K} \frac{|M(k)|}{H_{k,M(k)}} \left(2^{\left(\beta_{(t)} - \beta_k^{M(k)}\right)\gamma_k / M(k)} - 1 \right) + \sum_{k=1}^{K} \alpha_k^{M(k)}$$

These values are levels of the total power where additional subcarriers are removed for users *K* lists.

The complete solution of the problem is as follows:

1. If $P_{Total} \geq P_{least}$, solve the line search problem described earlier.
2. If $P_{(t)} > P_{Total} \geq P_{(t+1)}$, determine the which subcarriers are still active, *M(k)*, generated in **Step 3**, then solve the line search problem described earlier.

As an example, assuming that $K = 3$ and $N = 12$, the first user has subcarriers (1 to 4), the second user has subcarriers (5 to 8), and the third user has subcarriers (9 to 12), gammas are 1, 4, and 3, respectively. The gain matrix H as follow:

$$H = \begin{bmatrix} 1.487 & 1.520 & 1.489 & 1.391 & 1.010 & 0.527 & 0.234 & 0.037 & 0.033 & 0.322 & 0.846 & 1.300 \\ 4.058 & 3.869 & 2.945 & 1.913 & 1.573 & 1.607 & 1.509 & 1.530 & 1.750 & 2.348 & 3.195 & 3.758 \\ 3.300 & 2.984 & 1.941 & 1.169 & 0.930 & 0.881 & 1.006 & 1.205 & 1.384 & 1.887 & 2.438 & 2.873 \end{bmatrix}$$

Assuming that the available total power is sufficiently small. Then, applying the previous method to identify which subcarrier will be removed from the user's list. We obtained the result that is shown in Table (3-1).

Table 3-1 Removing order of Subcarriers

t	β	User	Subcarrier
1	0.7729	3	9
2	0.3256	3	10
3	0.3208	1	4
4	0.0789	3	11
5	0.0436	2	7
6	0.0342	1	1
7	0.0303	1	3
8	0.0255	2	8
9	0.0078	2	5

3.2.4 NUMERICAL EXAMPLE 1 FOR POWER ALLOCATION PROBLEM USING THE PROPOSED ALGORITHM (SMALL PROBLEM)

The H gain matrix (users vs. subcarriers) is given for 4 users and 8 subcarriers for problem. Moreover, the total power is 10. We would like to maximize the total capacity of the OFDMA system such that the fairness constraints are achieved. In this example, we code the proposed algorithm in MATLAB to solve the power allocation problem. The data and the result are shown in Table (3-2).

$$H = \begin{bmatrix} 73 & 36 & 90 & 42 & 77 & 97 & 34 & 22 \\ 73 & 40 & 77 & 90 & 48 & 72 & 25 & 11 \\ 6 & 1 & 72 & 1 & 96 & 14 & 37 & 2 \\ 38 & 94 & 39 & 3 & 10 & 46 & 85 & 62 \end{bmatrix}$$

Gammas (the proportional constants) for 4 users = [1.0000 0.3365 0.3411 0.9821], respectively.

$$Allocation = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Table 3-2 Results of the proposed algorithm (small problem)

Problem	(users vs subcarriers)	Total Capacity	Time (seconds)	Fairness Index
1	(4 vs. 8)	53.9613	0.004287	1

3.2.5 NUMERICAL EXAMPLE 2 FOR POWER ALLOCATION PROBLEM USING THE PROPOSED ALGORITHM (MEDIUM PROBLEM)

The H gain matrix (users vs. subcarriers) is given for 6 users and 16 subcarriers for problem. Moreover, the total power is 10. We would like to maximize the total capacity of the OFDMA system such that the fairness constraints are achieved. In this example, we code the proposed algorithm in MATLAB to solve the power allocation problem. The data and the result are shown in Table (3-3).

$$\boldsymbol{H} = \begin{bmatrix} 87 & 73 & 24 & 83 & 78 & 84 & 30 & 94 & 15 & 100 & 94 & 76 & 44 & 15 & 12 & 29 \\ 50 & 88 & 34 & 62 & 84 & 58 & 75 & 81 & 15 & 12 & 31 & 64 & 13 & 16 & 39 & 71 \\ 30 & 34 & 31 & 81 & 99 & 48 & 55 & 18 & 99 & 18 & 60 & 50 & 98 & 76 & 32 & 22 \\ 20 & 78 & 80 & 86 & 60 & 95 & 63 & 79 & 88 & 22 & 63 & 79 & 34 & 95 & 11 & 95 \\ 7 & 17 & 93 & 8 & 31 & 92 & 25 & 99 & 8 & 25 & 54 & 72 & 48 & 12 & 43 & 3 \\ 45 & 1 & 53 & 14 & 2 & 70 & 69 & 40 & 8 & 27 & 29 & 11 & 68 & 81 & 78 & 35 \end{bmatrix}$$

Gammas = [1.0000   0.6437   1.0131   1.6499   0.6709   0.3185], respectively.

$$Allocations = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Table 3-3 Results of the proposed algorithm (medium problem)

Problem	(users vs subcarriers)	Total Capacity	Time (seconds)	Fairness Index
2	(6 vs. 16)	93.6519	0.014346	1

3.2.6 NUMERICAL EXAMPLE 3 FOR POWER ALLOCATION PROBLEM USING THE PROPOSED ALGORITHM (LARGE PROBLEM)

The H gain matrix (users vs. subcarriers) is given for 10 users and 64 subcarriers for problem. Moreover, the total power is 10. We would like to maximize the total capacity of the OFDMA system such that the fairness constraints are achieved. In this example, we code the proposed algorithm in MATLAB to solve the power allocation problem. The data and the result are shown in Table (3-4).

Table 3-4 Results of the proposed algorithm (large problem)

Problem	(users vs subcarriers)	Total Capacity	Time (seconds)	Fairness Index
3	(10 vs. 64)	250.4039	0.114348	1

In this example 3 along with examples 1 and 2, we notice that the fairness index has a value of 1; which means that the Qos requirements for each user in the system are satisfied. The complete results for the previous examples are shown in Table (3-5) and the computational times are shown in Figure (3-2), in all the three examples, we can notice the computational times didn't exceed 1 second.

Table 3-5 Result of proposed algorithm for power allocation problem

Problem	(users vs subcarriers)	Total Capacity	Time (seconds)	Fairness Index
1	(4 vs. 8)	53.9613	0.004287	1
2	(6 vs. 16)	93.6519	0.014346	1
3	(10 vs. 64)	250.4039	0.114348	1

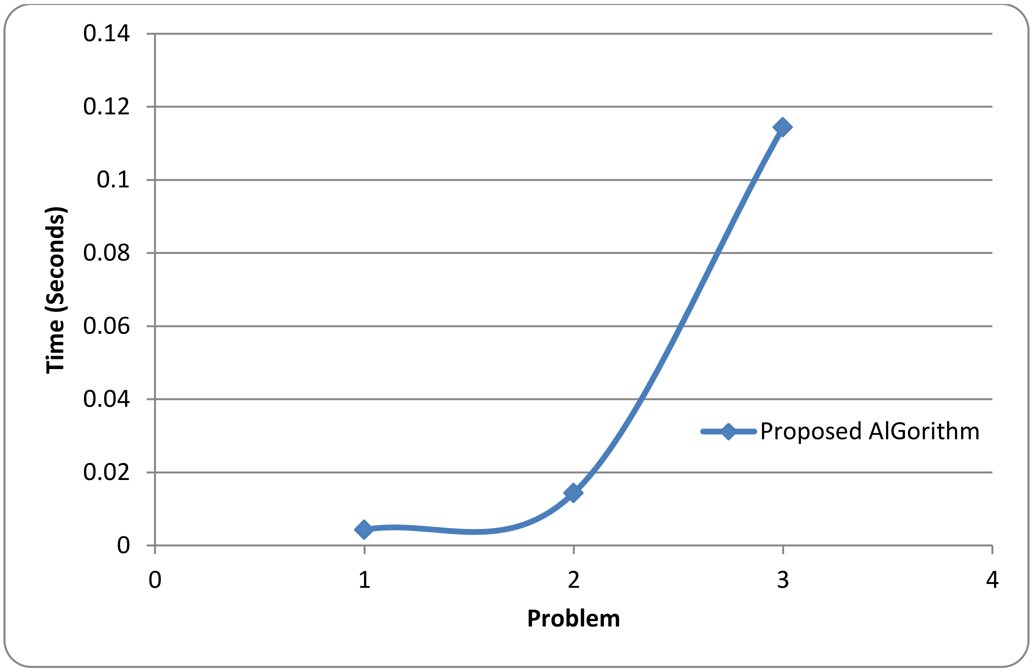

Figure 3-2 Computational Time for the problem using the proposed algorithm

3.2.7 NUMERICAL EXAMPLE 4 FOR POWER ALLOCATION PROBLEM USING LINGO 12 (SMALL PROBLEM)

The H gain matrix (users vs. subcarriers) is given for 4 users and 8 subcarriers for problem. Moreover, the total power is 10. We would like to maximize the total capacity of the OFDMA system such that the fairness constraints are achieved. In this example, we code the mathematical formulation of the power allocation problem using LINGO 12 to solve the problem. The data and the result are shown in Table (3-6).

$$H = \begin{bmatrix} 73 & 36 & 90 & 42 & 77 & 97 & 34 & 22 \\ 73 & 40 & 77 & 90 & 48 & 72 & 25 & 11 \\ 6 & 1 & 72 & 1 & 96 & 14 & 37 & 2 \\ 38 & 94 & 39 & 3 & 10 & 46 & 85 & 62 \end{bmatrix}$$

Gammas (the proportional constants) for 4 users = [1.0000 0.3365 0.3411 0.9821], respectively.

$$Allocation = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Table 3-6 Results of LINGO 12 for power allocation (small problem)

Problem	(users vs subcarriers)	Total Capacity	Time (seconds)	Fairness Index
1	(4 vs. 8)	53.9613	1.235	1

3.2.8 NUMERICAL EXAMPLE 5 FOR POWER ALLOCATION PROBLEM USING LINGO 12 (MEDIUM PROBLEM)

The H gain matrix (users vs. subcarriers) is given for 6 users and 16 subcarriers for problem. Moreover, the total power is 10. We would like to maximize the total capacity of the OFDMA system such that the fairness constraints are achieved. In this example, we code the mathematical formulation of the power allocation problem using LINGO 12 to solve the problem. The data and the result are shown in Table (3-7).

$$
H = \begin{bmatrix}
87 & 73 & 24 & 83 & 78 & 84 & 30 & 94 & 15 & 100 & 94 & 76 & 44 & 15 & 12 & 29 \\
50 & 88 & 34 & 62 & 84 & 58 & 75 & 81 & 15 & 12 & 31 & 64 & 13 & 16 & 39 & 71 \\
30 & 34 & 31 & 81 & 99 & 48 & 55 & 18 & 99 & 18 & 60 & 50 & 98 & 76 & 32 & 22 \\
20 & 78 & 80 & 86 & 60 & 95 & 63 & 79 & 88 & 22 & 63 & 79 & 34 & 95 & 11 & 95 \\
7 & 17 & 93 & 8 & 31 & 92 & 25 & 99 & 8 & 25 & 54 & 72 & 48 & 12 & 43 & 3 \\
45 & 1 & 53 & 14 & 2 & 70 & 69 & 40 & 8 & 27 & 29 & 11 & 68 & 81 & 78 & 35
\end{bmatrix}
$$

Gammas = [1.0000   0.6437   1.0131   1.6499   0.6709   0.3185], respectively.

$$
Allocations = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
$$

Table 3-7 Results of LINGO 12 for power allocation (medium problem)

Problem	(users vs subcarriers)	Total Capacity	Time (seconds)	Fairness Index
2	(6 vs. 16)	93.6519	3.471	1

3.2.9 NUMERICAL EXAMPLE 6 FOR POWER ALLOCATION PROBLEM USING LINGO 12 (LARGE PROBLEM)

The H gain matrix (users vs. subcarriers) is given for 10 users and 64 subcarriers for problem. Moreover, the total power is 10. We would like to maximize the total capacity of the OFDMA system such that the fairness constraints are achieved. In this example, we code the mathematical formulation of the power allocation problem using LINGO 12 to solve the problem. The data and the result are shown in Table (3-8).

Table 3-8 Results of LINGO 12 for power allocation (large problem)

Problem	(users vs subcarriers)	Total Capacity	Time (seconds)	Fairness Index
3	(10 vs. 64)	250.4039	7.269	1

In this example 4 along with examples 5 and 26, we notice that the fairness index has a value of 1; which means that the Qos requirements for each user in the system are satisfied while we notice also that the computational time increases when the number of users and subcarriers are increase as shown in Figure (3-3). The complete results for the previous examples are shown in Table (3-9).

Table 3-9 Results of LINGO 12 for power allocation

Problem	(users vs subcarriers)	Total Capacity	Time (seconds)	Fairness Index
1	(4 vs. 8)	53.9613	1.235	1
2	(6 vs. 16)	93.6519	3.471	1
3	(10 vs. 64)	250.4039	7.269	1

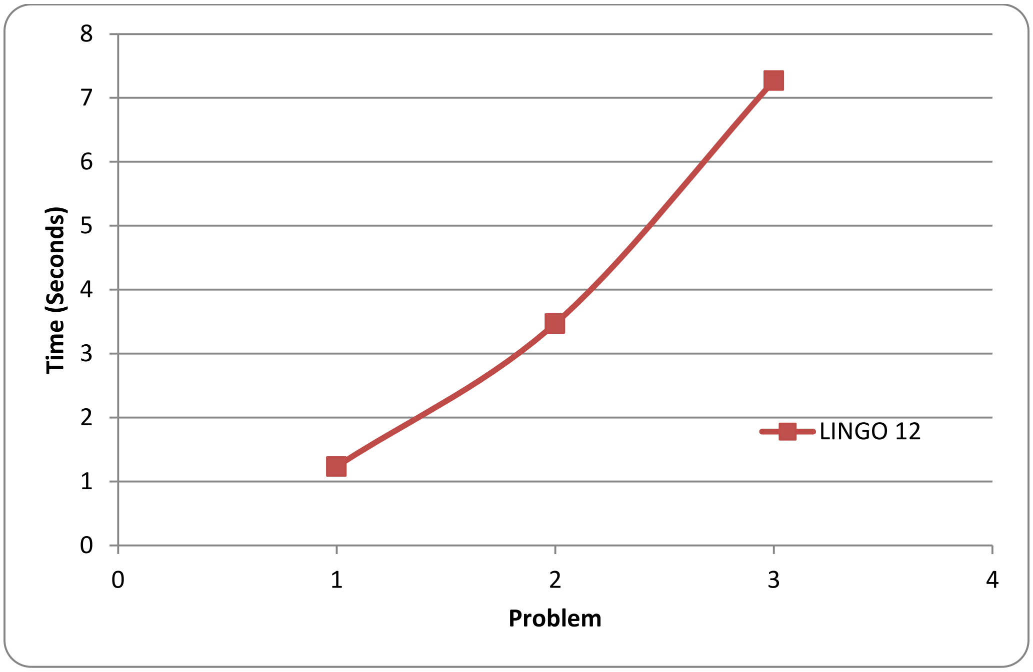

Figure 3-3 Computational Time for the problem using LINGO 12

In summary, the results show that when the number of users and subcarriers for the power allocation problem increase, the time it takes to solve the optimization problem increase in LINGO 12 while the proposed algorithm maintain the time of solving the optimization problem in less than one second as shown in Figure 3-4.

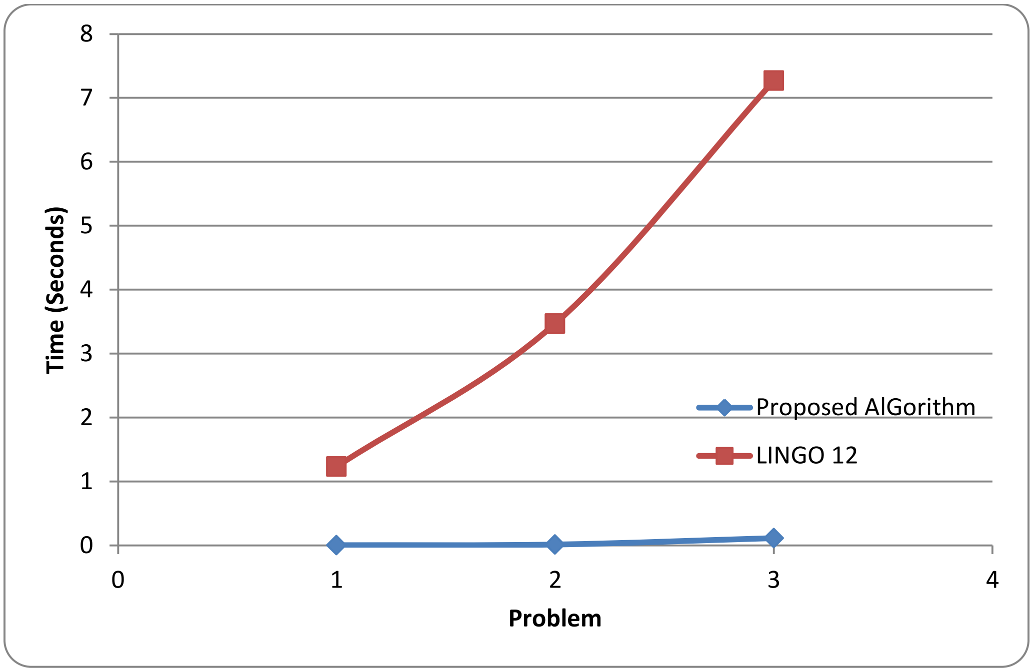

Figure 3-4 Computational Time for Power Allocation Problem

3.2.10 ADVANTAGES OF THE PROPOSED ALGORITHM

There are many advantages for the proposed algorithm. These are the following:

1. It can be used for any number of users and subcarriers.
2. Using a line search with only one variable to solve the power allocation problem.
3. Back substituting method.
4. Guaranteed the fairness constraint satisfied with 100%.
5. The computational time is very short.

3.3 MODEL II: POWER ALLOCATION PROBLEM WITH PARTIALLY RELAXED FAIRNESS CONSTRAINTS

In this section of the thesis, we consider the case of infeasible problem without removing and recalculating the values of βs. From the previous subsection (3.2.3), the fairness conditions may result in an infeasible solution problem if the total power available for allocation is not sufficiently large. We provide the condition for having feasible problem along with determine the power levels for the OFDMA system. Therefore, we propose a new formulation where we relax the fairness constraints using a controlled tolerance δ.

The objective of this model is to maximize the total capacity of the OFDMA system, i.e. the capacity for each user, with controlling tolerance for fairness condition. This means that the problem is to find the power allocation to the subcarriers that will maximize the total capacity of all users such that the weighted total power assigned to each user within a specific range of difference. The Nonlinear Program is described as follows:

1 – The objective function for Model II

As mentioned earlier in this chapter, the function to be optimized is the total capacity of the systems. In other words, it means the objective of this model is to maximize the capacity of each user and the objective function can be presented as follow

$$\text{Maximize} \sum_{k=1}^{K} \sum_{n \in N(k)} \log_2(1 + p_{k,n} H_{k,n}) \qquad (3.19)$$

where

K	is the number of users
$N(k)$	is the set of sub-carriers assigned to user k, $k = 1,2,\ldots,K$
$p_{k,n}$	is the power allocated to user k on the subcarrier n
$H_{k,n}$	is the channel power gain for user k in subcarrier n

It is noticeable that the objective function of the model is a logarithmic function; that is a concave function.

2 – The constraints for Model I

The problem has four types of constraints that bound the feasible region of the solution. These are the following:

1. The total power constraint; the total power should not exceed available power

$$\sum_{k=1}^{K} \sum_{n \in N(k)} p_{k,n} = P_{Total} \qquad (3.20)$$

   where P_{Total} is the total available bandwidth and power. It is a straight forward to show that this constant is satisfied as equality at an optimal solution.

2. The set of fairness constraints: adding a tolerance δ constant in order to expand the range of Y.

$$-\delta \leq \frac{\sum_{n\in N(k)} \log_2\left(1 + p_{k,n}H_{k,n}\right)}{\gamma_k} - Y \leq \delta, \quad 1 \leq k \leq K \tag{3.21}$$

Where Y is a variable to be determined and for simplicity we separate (3.21) as follows

$$\frac{\sum_{n\in N(k)} \log_2\left(1 + p_{k,n}H_{k,n}\right)}{\gamma_k} - Y \leq \delta, \quad 1 \leq k \leq K \tag{3.22}$$

$$Y - \frac{\sum_{n\in N(k)} \log_2\left(1 + p_{k,n}H_{k,n}\right)}{\gamma_k} \leq \delta, \quad 1 \leq k \leq K \tag{3.23}$$

3. The set of non-negativity constraints; the allocated power to each subcarrier must be nonnegative in order to satisfy the total power constraint in the systems as we defined earlier and the non-negativity constraints can be represented and modeled as follow

$$p_{k,n} \geq 0, \quad n = 1,2,\dots,N \tag{3.24}$$

In other words, the complete and the standard form of the optimization problem of power allocation with relax fairness constraints is shown in Figure 3-5.

$$Maximize \qquad \sum_{k=1}^{K} \sum_{n \in N(k)} \log_2(1 + p_{k,n} H_{k,n})$$

$$Subject\ to$$

$$\sum_{k=1}^{K} \sum_{n \in N(k)} p_{k,n} \leq P_{Total}$$

$$\frac{\sum_{n \in N(k)} \log_2(1 + p_{k,n} H_{k,n})}{\gamma_k} - Y \leq \delta$$

$$Y - \frac{\sum_{n \in N(k)} \log_2(1 + p_{k,n} H_{k,n})}{\gamma_k} \leq \delta$$

$$p_{k,n} \geq 0$$

Figure 3-5 The power allocation optimization model with relaxed fairness

3.3.1 MODEL II ANALYSIS

A controlled tolerance δ is a known value that expands the feasibility region for variable Y in case of having insufficient total power for solving the optimization problem. That means the total capacity of the system increase and the fairness among K users decrease as the value of δ increase. Accordingly, specifying the range of possible values for δ in order to keep the fairness among K users as high as possible by maximizing the total capacity of the OFDMA system. Clearly, the value of δ must be a non-negative value $\delta \geq 0$.

To compute the possible values of δ, we arrange the capacity of each user k in an ascending order on the number line as shown in Figure 3-2. Then, we assume that $G_k = \frac{R_k}{\gamma_k}$, the smallest amount of $\frac{R_k}{\gamma_k}$ denoted as G_{min} and the largest amount of $\frac{R_k}{\gamma_k}$ denoted as G_{max}, for $1 \leq k \leq K$.

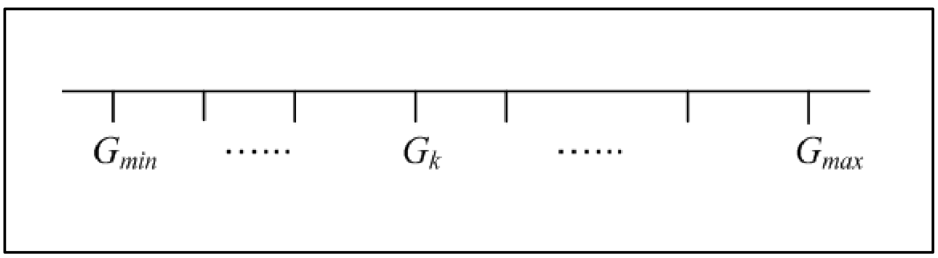

Figure 3-6 Values on line number

If δ = 0, then $G_k = Y$ as MODEL I discussed in the previous section (3.2) as shown in Figure 3-3.

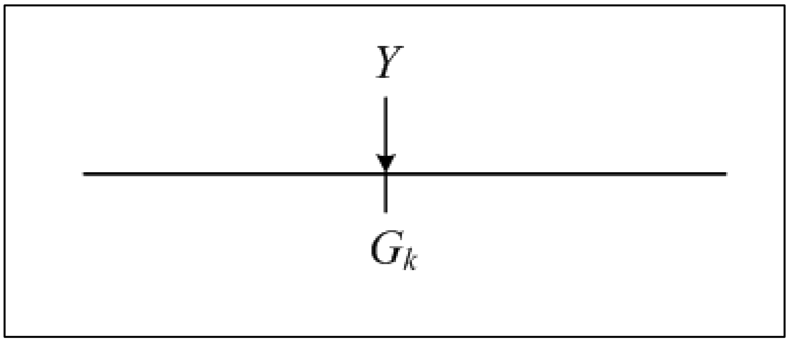

Figure 3-7 All values are equal - Model I

If $\delta > 0$, then according to the (3.22) and (3.23), we obtain the following inequalities

$$Y \leq G_k + \delta \tag{3.25}$$

$$Y \geq G_k - \delta \tag{3.26}$$

To satisfy the above constraints (3.25) and (3.25) such that the smallest value of δ will not affect the G_{min} and G_{max} (Figure 3-4), we equate the following to get the value of δ

$$Y \leq G_{max} + \delta \tag{3.27}$$

$$Y \geq G_{min} - \delta \tag{3.28}$$

$$Y = \frac{G_{max} - G_{min}}{2} \tag{3.29}$$

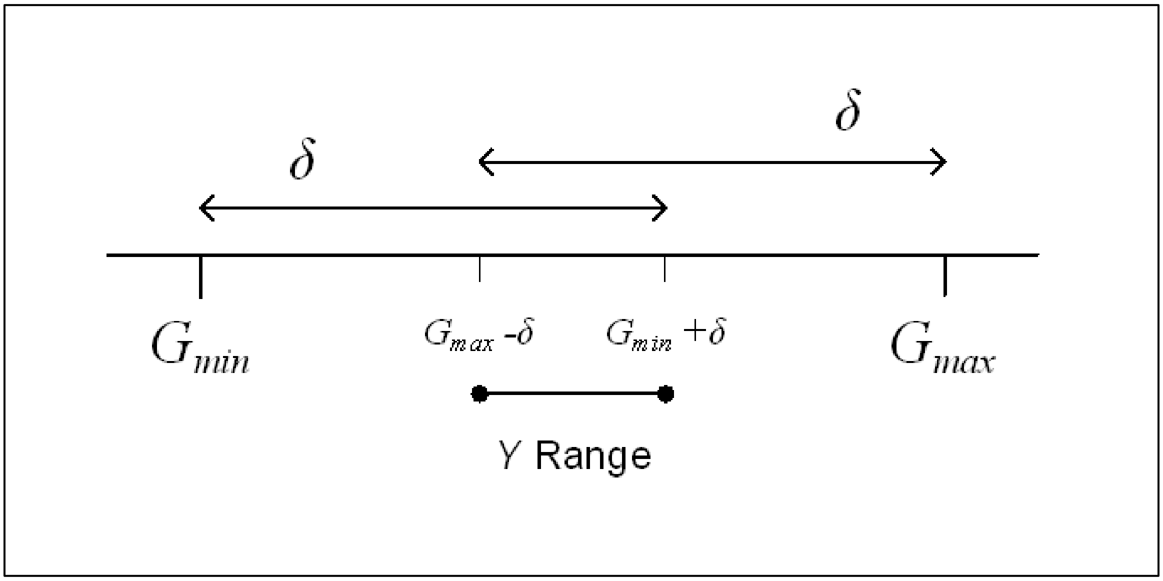

Figure 3-8 The range of Y value

Obviously, the value of *Y* in that range will be in the middle

$$\delta = \frac{G_{max} + G_{min}}{2} \tag{3.30}$$

3.3.2 NUMERICAL EXAMPLE OF POWER ALLOCATION PROBLEM WITH REALXED CONSTRAINTS USING LINO 12

LINGO12 is used to implement the mathematical model, as an example using H for five users, ten subcarriers, and tolerance constant δ starts from 0 with 0.001 increments. The result as shown in Figures 3-9 and 3-10 shows that increasing the value of delta δ, the objective function value will increase. On the other hand, increasing the value of delta δ, the fairness index will decrease.

$$H = \begin{bmatrix} 11067.3 & 9301.6 & 5071.1 & 12233.8 & 12693.8 & 7304.1 & 5422.4 & 2568.6 & 5348.0 & 1933 \\ 41371.3 & 38219.2 & 53321.1 & 62352.6 & 56917.2 & 90844.4 & 133234.3 & 121879.5 & 147176.7 & 83394.2 \\ 65.4 & 60.7 & 42.4 & 68.6 & 68.2 & 58.8 & 35.9 & 12.0 & 14.58 & 24.3 \\ 42315.7 & 16619.2 & 3150.3 & 29680.6 & 44571.9 & 14389.07 & 11541.9 & 9637.2 & 9059.6 & 8192.9 \\ 967.9 & 1634.9 & 1868.5 & 1200.2 & 850.9 & 1921.5 & 2520.2 & 2589.03 & 2833.3 & 1823.52 \end{bmatrix}$$

$$Allocation = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

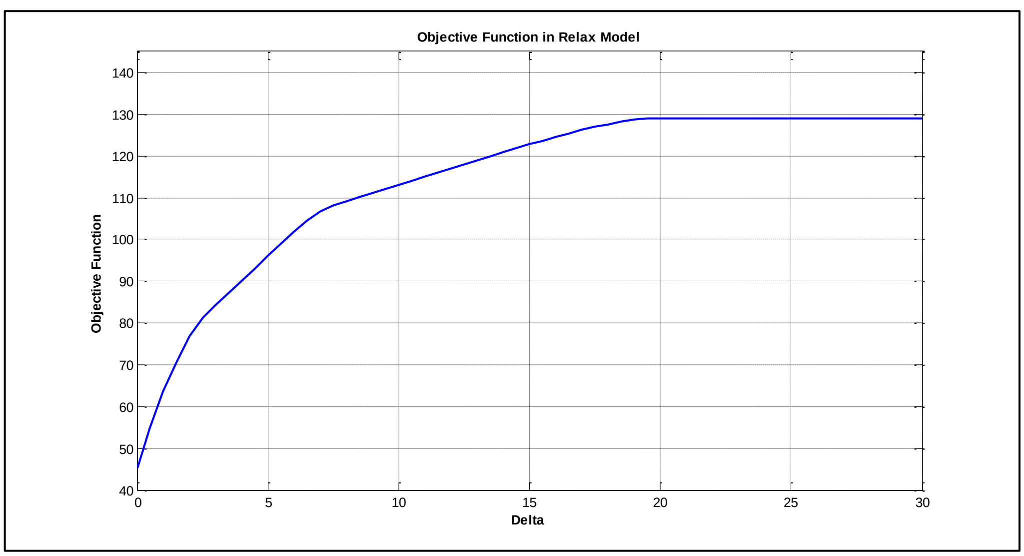

Figure 3-9 Objective Function Evaluation Model II

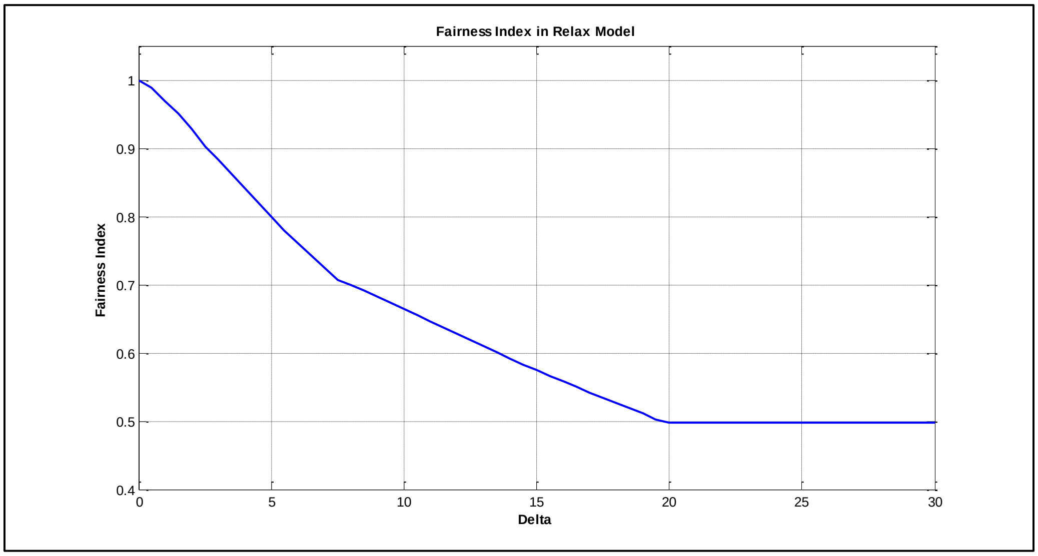

Figure 3-10 Fairness Index Evaluation Model II

Chapter 4

THE POWER AND SUBCARRIER ALLOCATION MODEL

4.1 INTRODUCTION

In this chapter, we discuss the model of allocating N sub-carriers to K users. This type of optimization problem is classified as Integer Program since all decision variables are bounded variables with lower bound 0, and upper bound 1. In fact, as we explained earlier the optimization problem consists of two types of programs; i.e. Nonlinear Program and Integer Program. Usually, the combination of both is called Mixed Integer Nonlinear Program (MINLP). Since we are dealing with variables that take values 0 or 1 only, it is possible to enumerate all the possible solutions one by one and evaluate each one of them separately, and then identifying the best solution encountered as the optimum one for the problem. But unfortunately, in real world applications such as our problem, the number of possibilities to check tends to be very large, that even if we use the most sophisticated computers available nowadays, the solution to such a problem cannot be obtained by total enumeration within the lifetime, making it impractical way to handle these types of optimization problem.

In this regard, we introduce an algorithm called Simulated Annealing (SA) to solve the problem. Also, we solve the model using LINGO 12 package.

4.2 SIMULATED ANNEALING HEURISTIC ALGORITHM

The word heuristic comes from the Greek word heuriskein that mean the art of problem solving. The heuristic methods and algorithms are usually applied to search for the good solutions within the feasible region in intelligent way. There are many types of heuristic search methods in the literature. One of the methods called Simulated Annealing method for searching to local optima solution for an optimization problem involving some random components in the way of proceeding the method.

Simulated Annealing (SA) was introduced first by S. Kirkpatrick et al. in 1983 [13]. The idea of SA is based on the statistical principle mechanism. In manufacturing, the annealing process requires heating and slowly cooling, such as substance in manufacturing processes, to obtain a strong shape structure. Indeed, the strength of the structure depends on the rate of cooling process. If the initial set up of the temperature is not sufficiently high enough or applying fast cooling process, imperfect structure is obtained. Hence, strong structure is grown from slow and careful cooling rate.

The SA is a stochastic algorithm which simulates the objective changes in a system subjected to a cooling process until it converges to steady state by creating an initial solution randomly. The general steps of SA are described as follows:

1. Set up a cooling schedule (setup of initial temperature and minimum temperature).

   It defines the temperature for each step in the SA algorithm. In fact, the algorithm performance is very sensitive to the cooling schedule choice. It has an essential role in the efficiency and the effectiveness of SA algorithm.

2. Generate an initial solution $s = s_{initial}$ and starting the algorithm at initial temperature.
3. Generate a random neighborhood solution s^*.
4. Calculate the difference between the new objective of step 3 and the objective of step 2. If the objective is improved, then $s = s^*$. Else, accept s^* with a probability of $e^{-\frac{the\ diffeence}{temperature}}$. The probability of acceptance is the main key of the SA algorithm which enables non-improving neighborhood solution to be selected.
5. Repeat steps 3 and 4 until the steady state zone is reached. For instance, number of rejected solutions (no improvement in the objective value) at each temperature.
6. Update the temperature.

   There are many ways and methods for updating.

7. Stop criteria is satisfied and best solution is found.

The SA steps for our optimization problem are described as follows:

1. Setup the cooling schedule.

   The initial temperature T_{old} is set to (500, 750, 1000) degrees (Experimentally found). Then, the temperature is decreased gradually in each level, using

   $T_{new} = \alpha\ T_{old}$ as updating formula where α is 0.9, in such a way that there is always a compromise between the cooling rate and the quality of the solution. Also, we set a low temperature as T_{low} = 3 degrees.

2. Generate a random solution (distributing all subcarriers to users randomly).
3. Generate a random neighborhood solution by swapping subcarriers among users one at a time with a probability of 0.75 or transferring a subcarrier from a user to another.
4. Calculate the difference between the new objective of step 3 and the objective of step 2. If the objective is improved, then $s = s^*$. Else, accept s^* with a probability of $e^{-\frac{the\ diffeence}{temperature}}$.
5. Repeat steps 3 and 4 until the steady state zone which is when the number of rejected solutions (1000, 1500, and 2000) times at each temperature.
6. Update the temperature. ($T_{new} = 0.9\ T_{old}$)
7. Stop when the criteria of stopping the algorithm is achieved, when $T_{new} < T_{low}$, is satisfied and best solution is found.

The attraction of the SA algorithm for our problem is that it is efficient enough to get optimum solution in a very little time comparing to optimization software packages since we have the power allocation part solved completely (discussed in the previous chapter) along with guaranteed that the fairness constraint is met and the fairness index is always perfect.

4.3 SOLUTION OF MODEL II USING LINGO 12

LINGO 12 is an optimization software package. Our optimization model is coded to fit the LINGO 12. Indeed, we provide an iterative method using the software by assuming a random initial solution to allocate powers for each subcarrier and then use these powers as an input to the optimization problem to obtain the allocations of subcarriers for each user k. Finally, we use LINGO 12 to obtain the subcarriers and powers allocations as one optimization problem.

4.3.1 ITERATIVE METHOD PROCEDURE

Step 1: Generate initial solution by assigning subcarriers N to K users to obtain powers allocated to each subcarrier using model I (power allocation problem).

Step 2: Use the obtained powers allocation as an input to the optimization problem in order to get subcarrier allocation to users.

Step 3: Again, use the obtained allocated subcarrier and use them as an input to the optimization problem in order to get power allocation for each subcarrier.

Step 4: Repeat steps 2 and 3 till no changes and improvement to our objective function.

Step 5: best allocations for powers and subcarriers allocations.

4.4 NUMERICAL EXAMPLE 1 USING SA ALGORITHM (SMALL PROBLEM)

Given H gain matrix (users vs. subcarriers), that has 4 users and 8 subcarriers. The total power (total power budget) is 10. We would like to maximize the total capacity of the OFDMA system such that the fairness constraints are achieved. In this example, we use Simulated Annealing algorithm to solve exactly the same problem 5 times with different initial solutions provided randomly at each different starting temperature.

At starting temperature 500 and the number of rejected points is 1000. We obtain the following results as shown in Table 4-1.

Table 4-1 Results of small size problem at temperature 500 and 1000 rejected points

Trial	Total Capacity	Time (seconds)	Optimal
1	53.9613	80.430048	53.9613
2	53.9613	80.107070	53.9613
3	53.9613	80.275745	53.9613
4	53.9613	80.739023	53.9613
5	53.9613	80.226533	53.9613

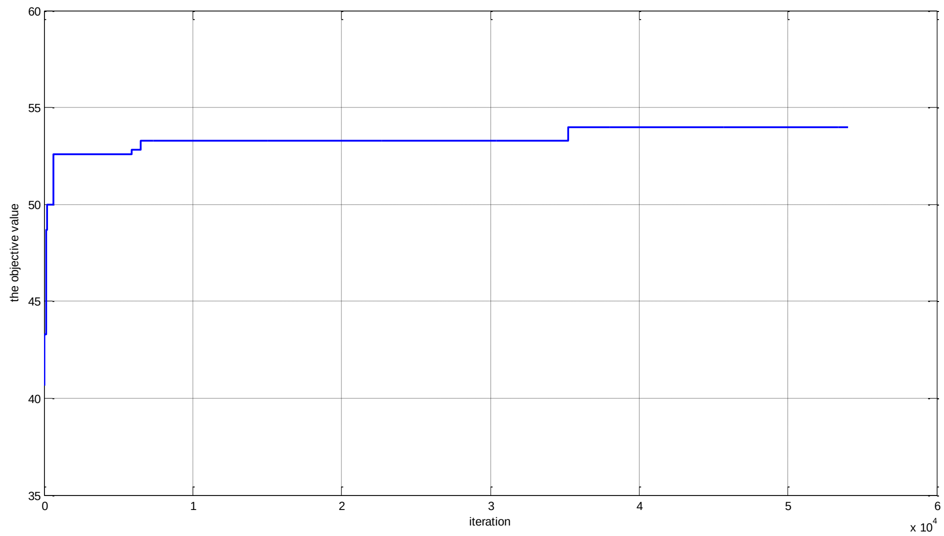

Figure 4-1 Trial 1 - SA Performance (small problem) - Temp 500, 1000 rejection

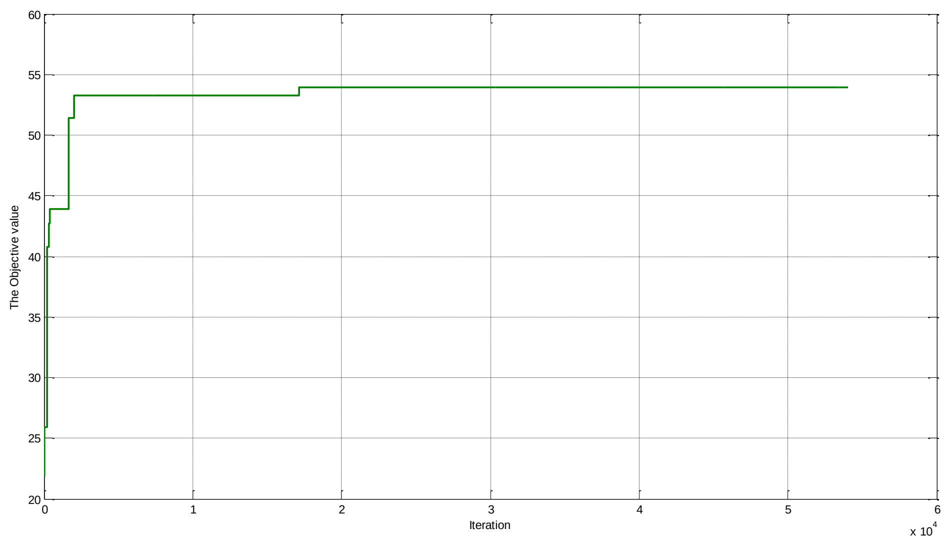

Figure 4-2 Trial 2 - SA Performance (small problem) - Temp 500, 1000 rejection

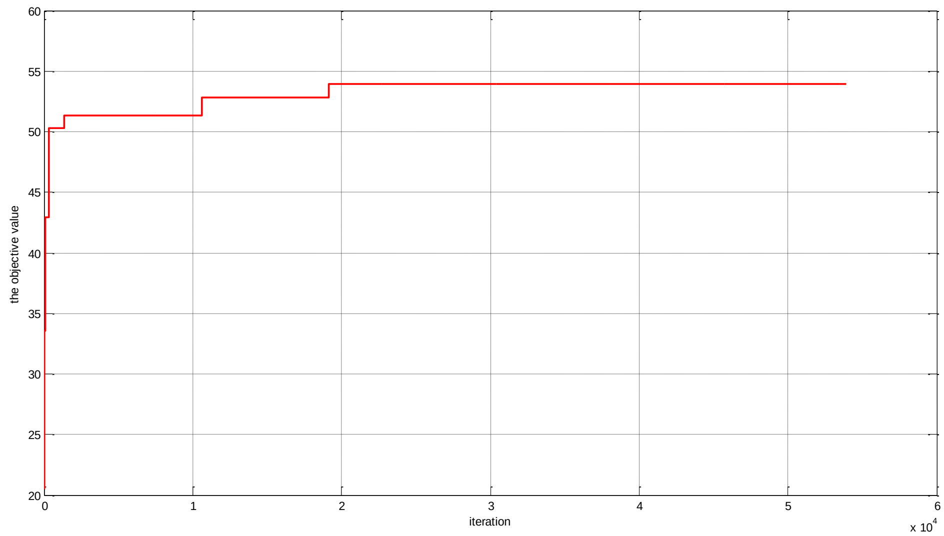

Figure 4-3 Trial 3 - SA Performance (small problem) - Temp 500, 1000 rejection

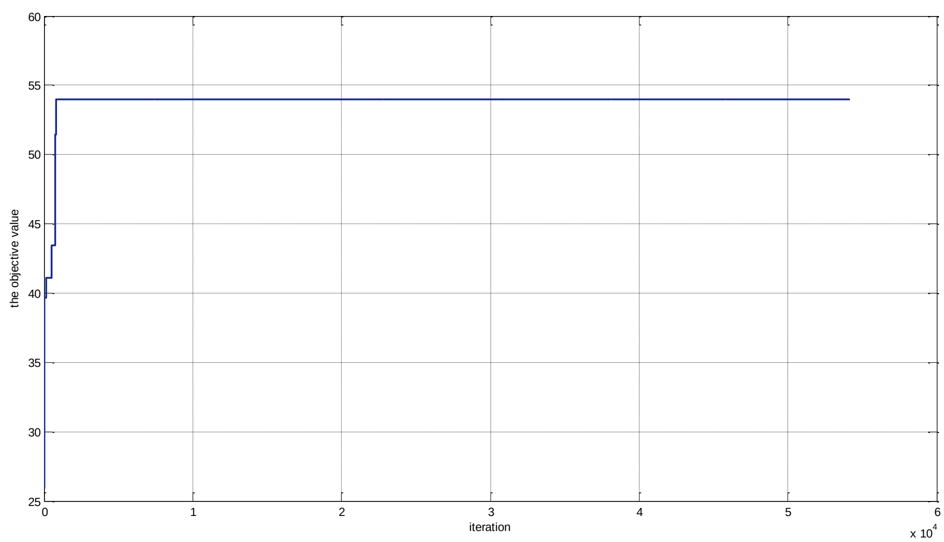

Figure 4-4 Trial 4 - SA Performance (small problem) - Temp 500, 1000 rejection

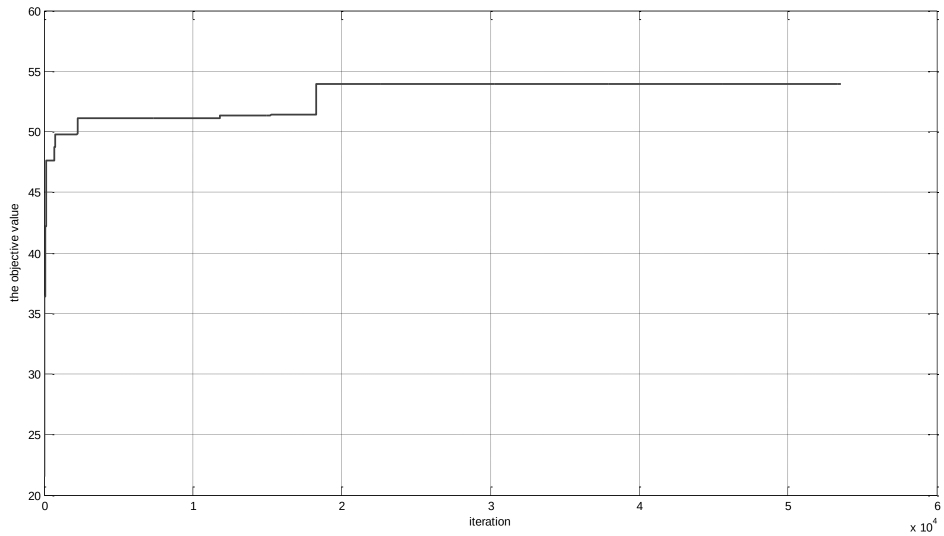

Figure 4-5 Trial 5 - SA Performance (small problem) - Temp 500, 1000 rejection

At starting temperature 750 and the number of rejected points is 1500. We obtain the following results as shown in Table 4-2.

Table 4-2 Results of small size problem at temperature 750 and 1500 rejected points

Trial	Total Capacity	Time (seconds)	Optimal
1	53.9613	128.887549	53.9613
2	53.9613	149.687687	53.9613
3	53.9613	153.749541	53.9613
4	53.9613	149.233602	53.9613
5	53.9613	147.965089	53.9613

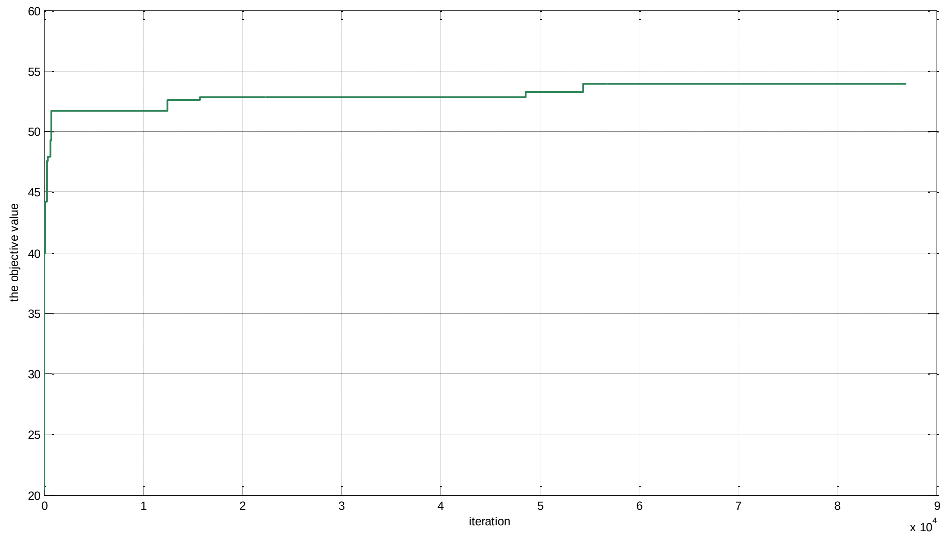

Figure 4-6 Trial 1 - SA Performance (small problem) - Temp 750, 1500 rejection

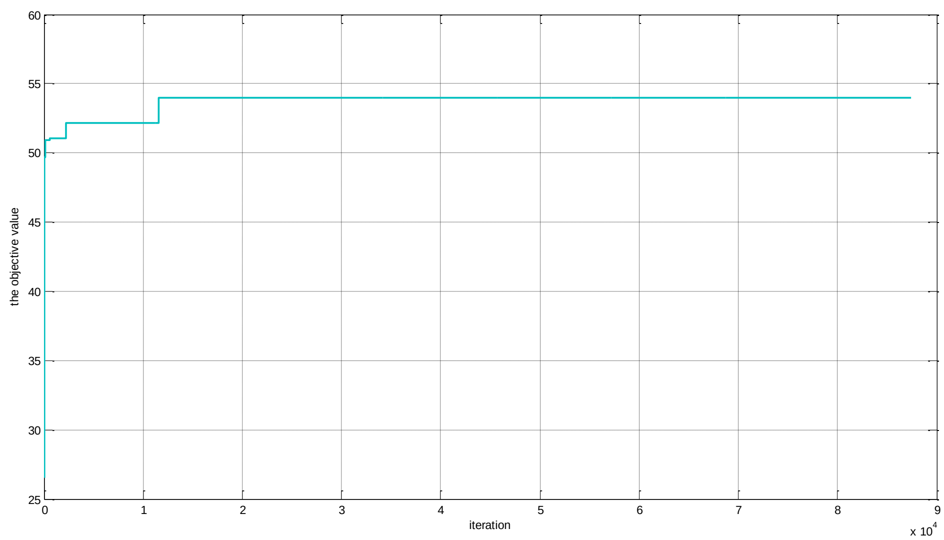

Figure 4-7 Trial 2 - SA Performance (small problem) - Temp 750, 1500 rejection

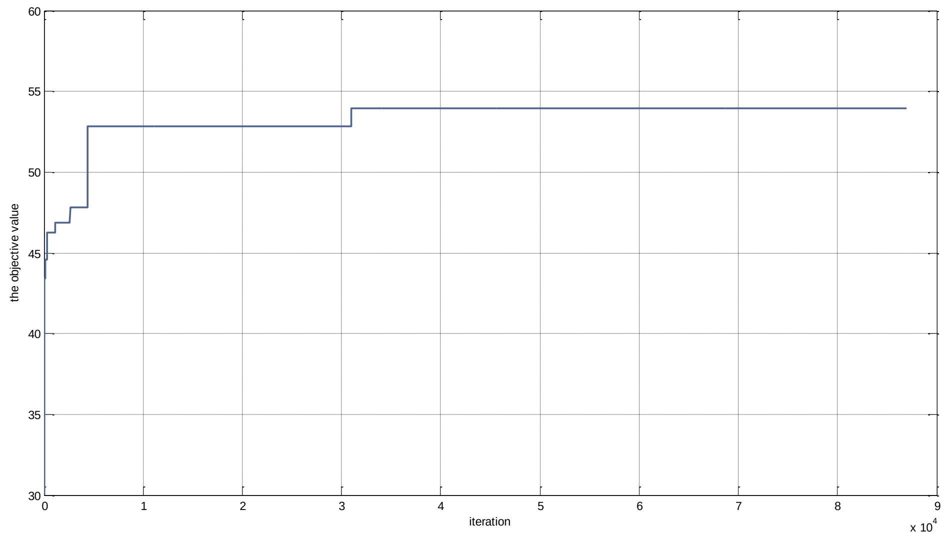

Figure 4-8 Trial 3 - SA Performance (small problem) - Temp 750, 1500 rejection

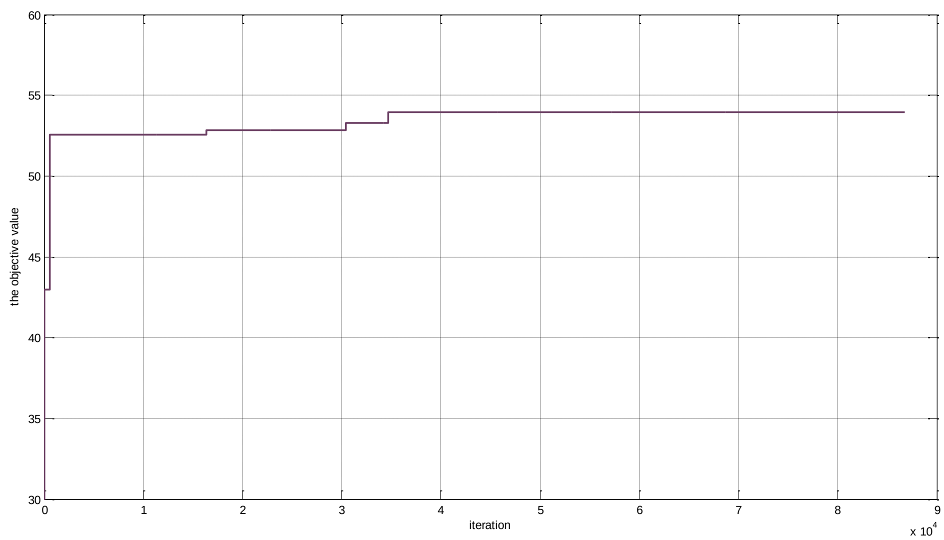

Figure 4-9 Trial 4 - SA Performance (small problem) - Temp 750, 1500 rejection

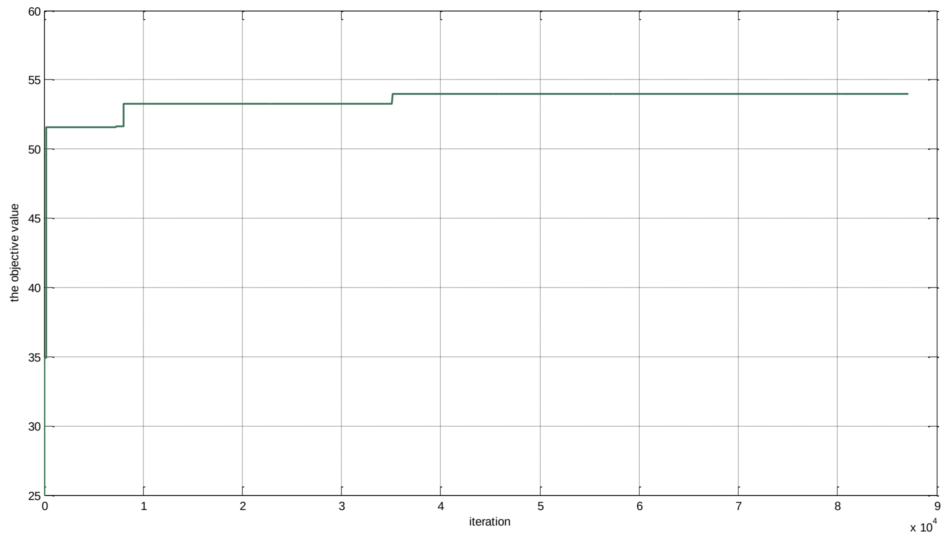

Figure 4-10 Trial 5 - SA Performance (small problem) - Temp 750, 1500 rejection

At starting temperature 1000 and the number of rejected points is 2000. We obtain the following results as shown in Table 4-3.

Table 4-3 Results of small size problem at temperature 1000 and 2000 rejected points

Trial	Total Capacity	Time (seconds)	Optimal
1	53.9613	207.552255	53.9613
2	53.9613	208.806516	53.9613
3	53.9613	209.054045	53.9613
4	53.9613	208.432596	53.9613
5	53.9613	217.667497	53.9613

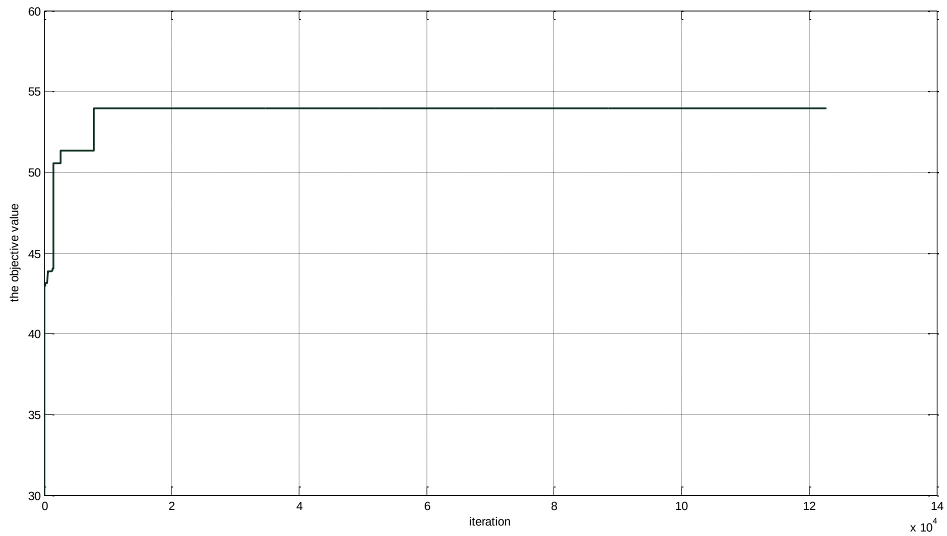

Figure 4-11 Trial 1 - SA Performance (small problem) - Temp 1000, 2000 rejection

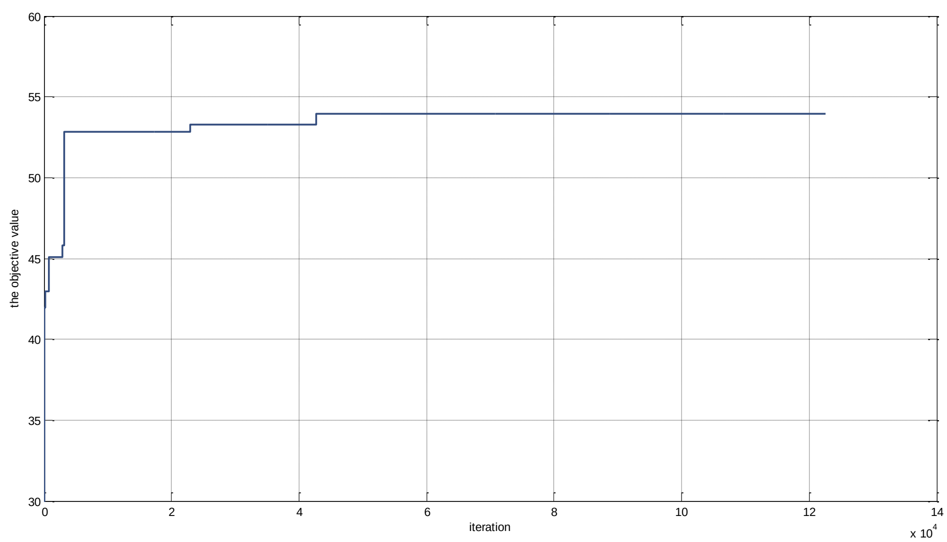

Figure 4-12 Trial 2 - SA Performance (small problem) - Temp 1000, 2000 rejection

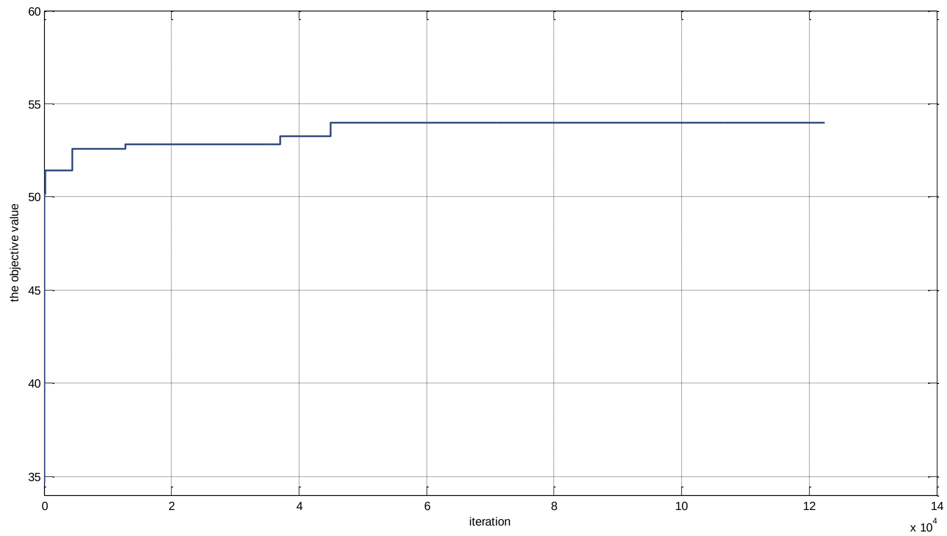

Figure 4-13 Trial 3 - SA Performance (small problem) - Temp 1000, 2000 rejection

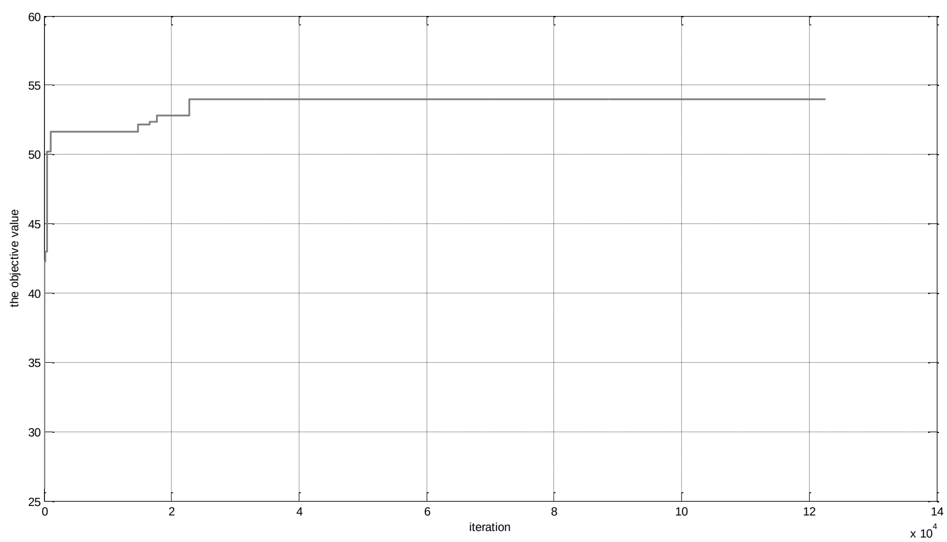

Figure 4-14 Trial 4 - SA Performance (small problem) - Temp 1000, 2000 rejection

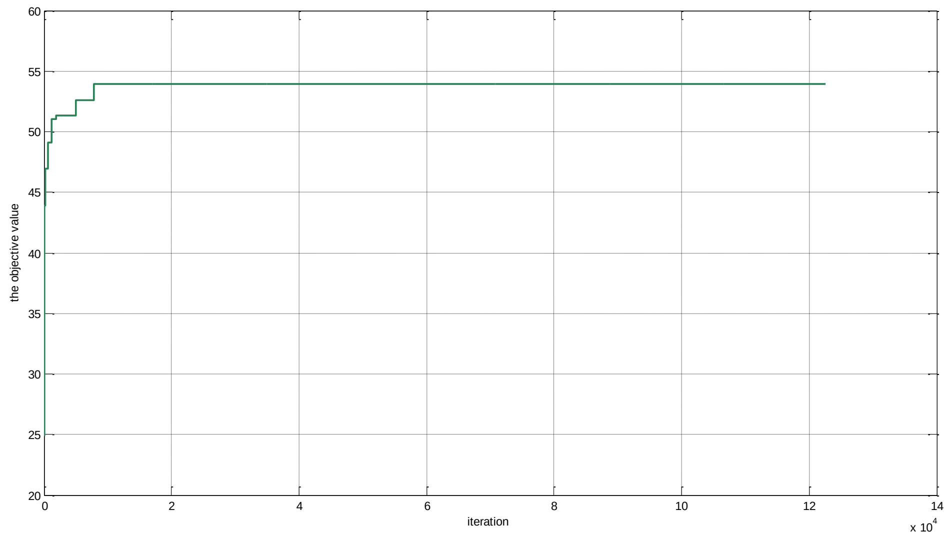

Figure 4-15 Trial 5 - SA Performance (small problem) - Temp 1000, 2000 rejection

4.5 NUMERICAL EXAMPLE 2 USING SA ALGORITHM (MEDIUM PROBLEM)

Given H gain matrix (users vs. subcarriers), that has 6 users and 16 subcarriers. The total power (total power budget) is 10. We would like to maximize the total capacity of the OFDMA system such that the fairness constraints are achieved. In this example, we use Simulated Annealing algorithm to solve exactly the same problem 5 times with different initial solutions provided randomly at each different starting temperature.

At starting temperature 500 and the number of rejected points is 1000. We obtain the following results as shown in Table 4-4.

Table 4-4 Results of medium size problem at temperature 500 and 1000 rejected

Trial	Total Capacity	Time (seconds)	Optimal
1	88.125656	104.458864	93.6519
2	86.762802	94.087176	93.6519
3	89.739006	89.739006	93.6519
4	89.768422	87.822627	93.6519
5	85.690908	88.232388	93.6519

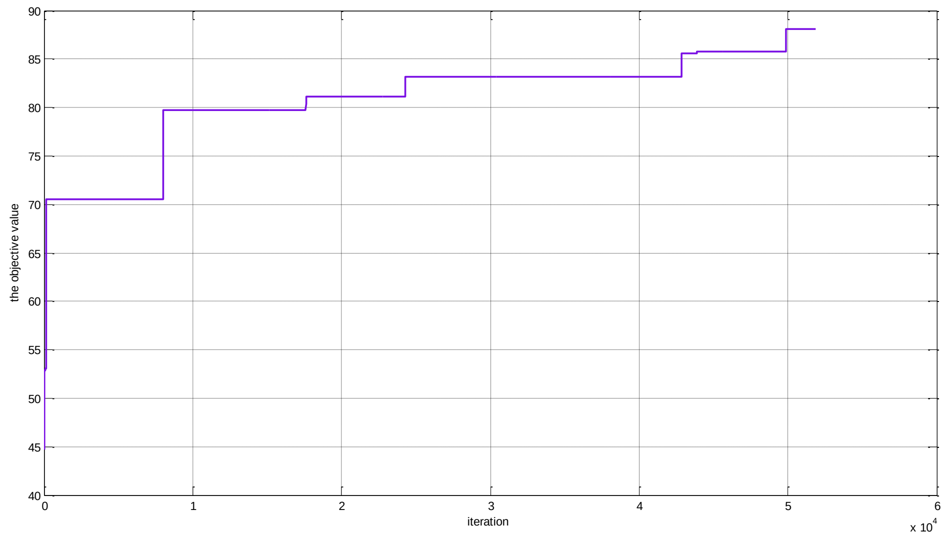

Figure 4-16 Trial 1 - SA Performance (medium problem) - Temp 500, 1000 rejection

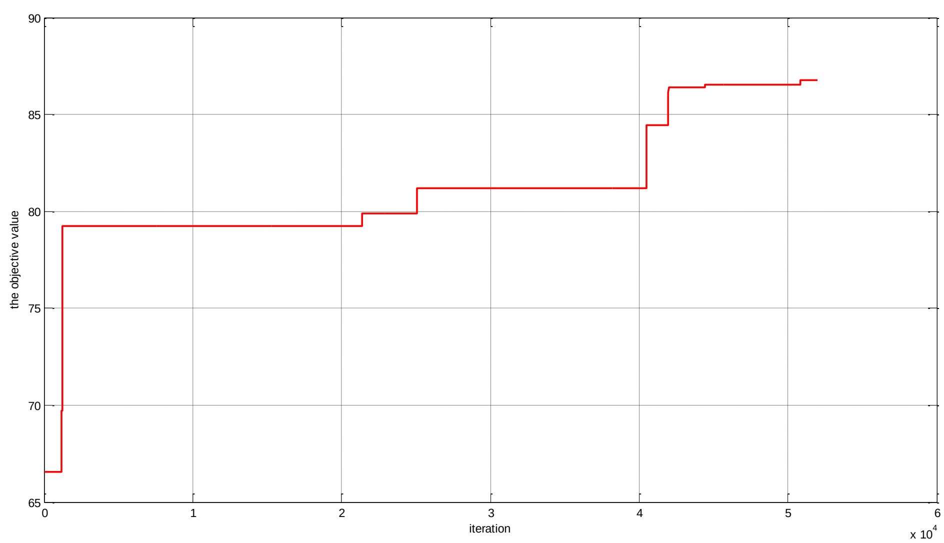

Figure 4-17 Trial 2 - SA Performance (medium problem) - Temp 500, 1000 rejection

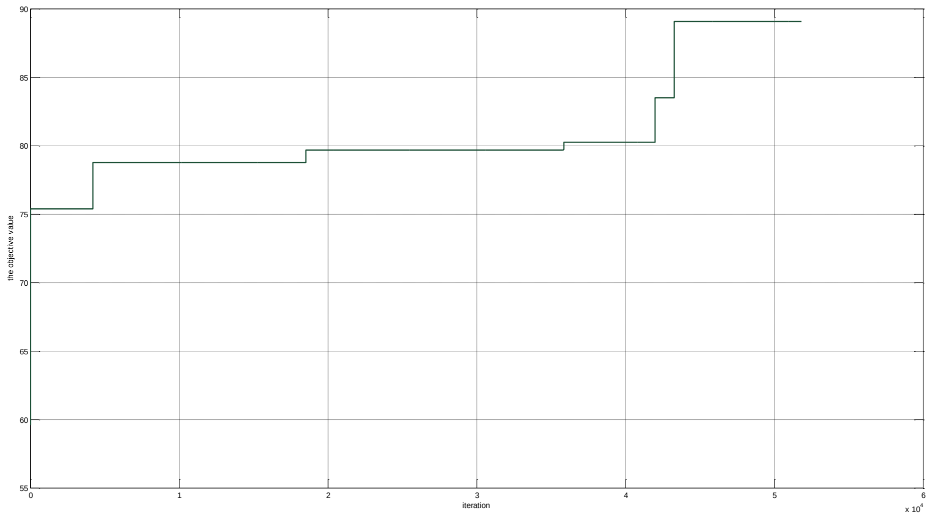

Figure 4-18 Trial 3 - SA Performance (medium problem) - Temp 500, 1000 rejection

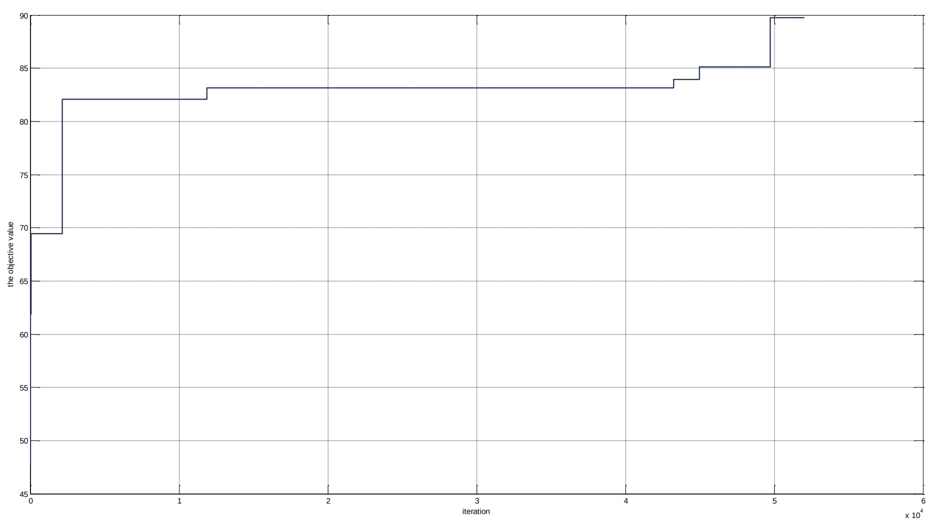

Figure 4-19 Trial 4 - SA Performance (medium problem) - Temp 500, 1000 rejection

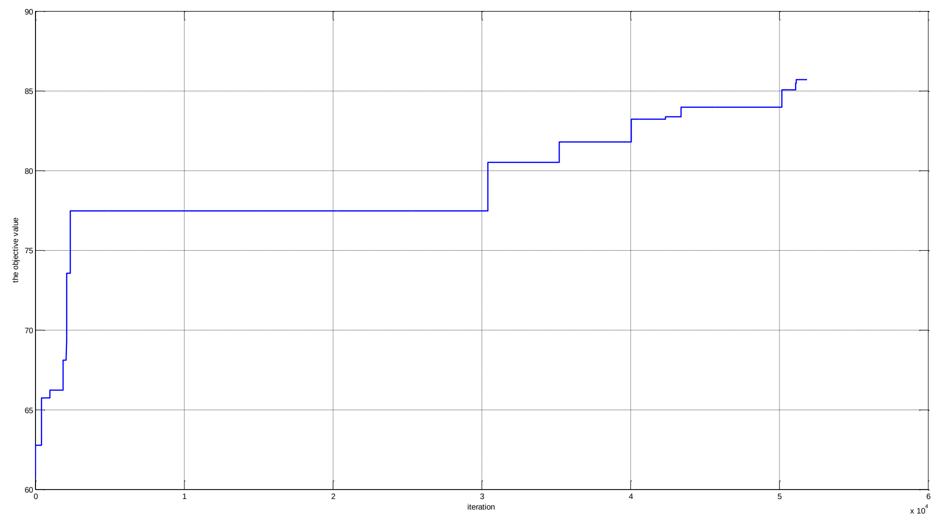

Figure 4-20 Trial 5 - SA Performance (medium problem) - Temp 500, 1000 rejection

At starting temperature 750 and the number of rejected points is 1500. We obtain the following results as shown in Table 4-5.

Table 4-5 Results of medium size problem at temperature 750 and 1500 rejected

Trial	Total Capacity	Time (seconds)	Optimal
1	90.413448	144.61433	93.6519
2	87.939269	140.07981	93.6519
3	91.895412	141.954471	93.6519
4	88.763500	141.004437	93.6519
5	87.784832	140.961187	93.6519

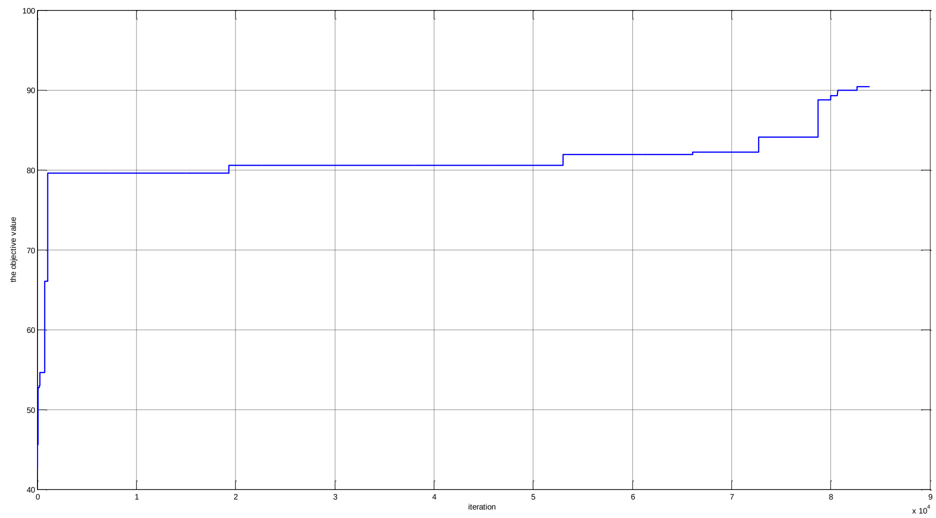

Figure 4-21 Trial 1 - SA Performance (medium problem) - Temp 750, 1500 rejection

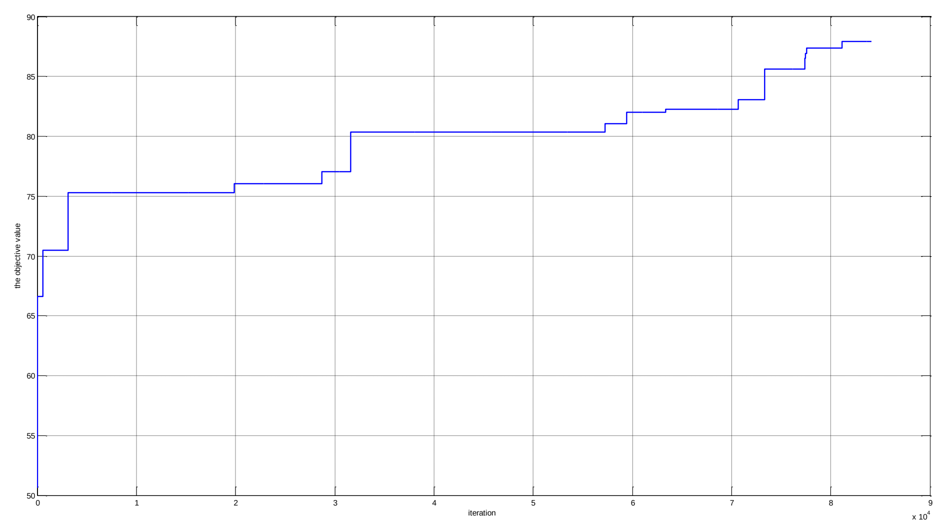

Figure 4-22 Trial 2 - SA Performance (medium problem) - Temp 750, 1500 rejection

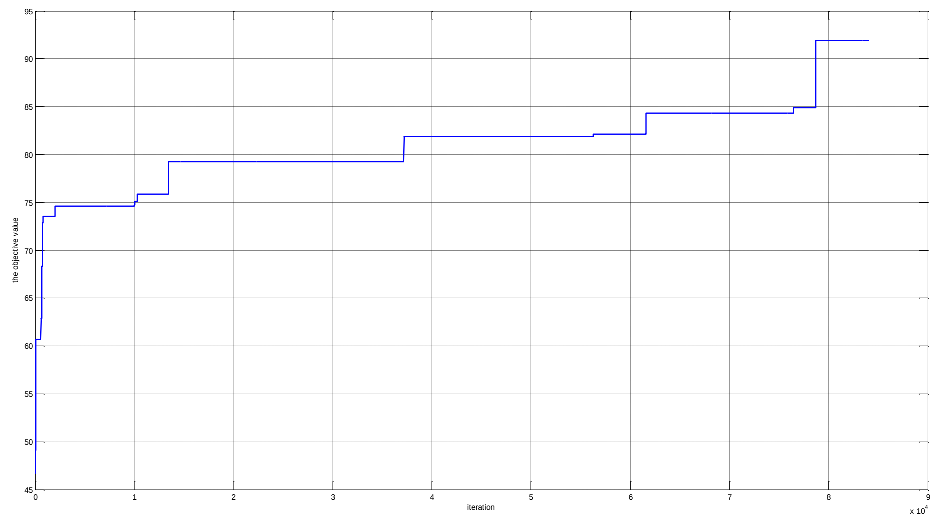

Figure 4-23 Trial 3 - SA Performance (medium problem) - Temp 750, 1500 rejection

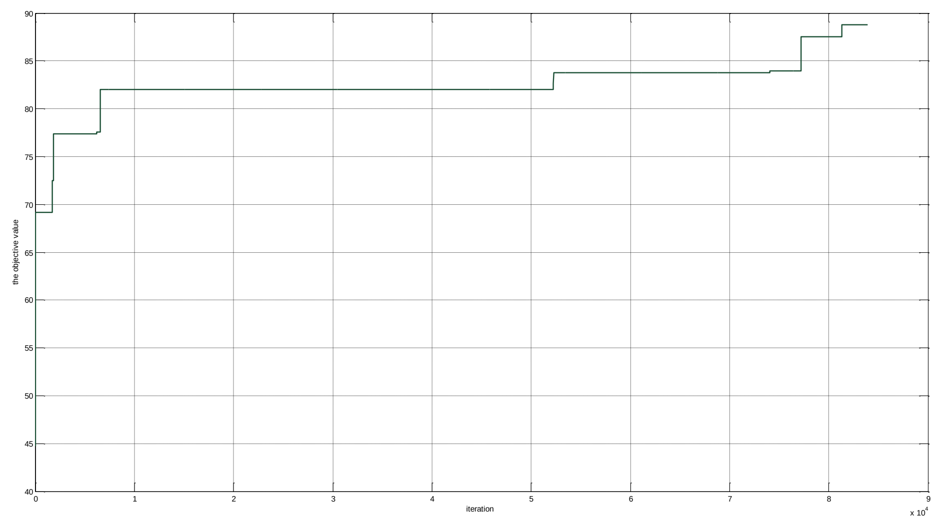

Figure 4-24 Trial 4 - SA Performance (medium problem) - Temp 750, 1500 rejection

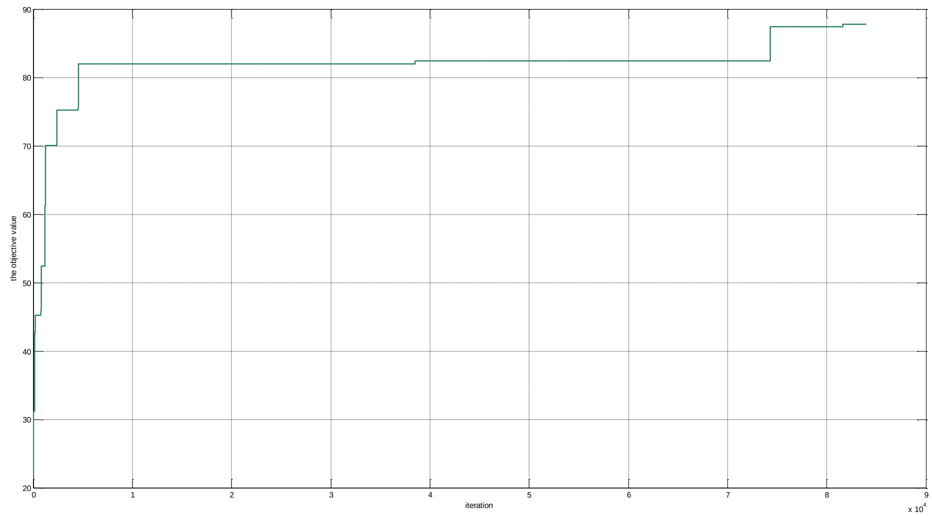

Figure 4-25 Trial 5 - SA Performance (medium problem) - Temp 750, 1500 rejection

At starting temperature 1000 and the number of rejected points is 2000. We obtain the following results as shown in Table 4-6.

Table 4-6 Results of medium size problem at temperature 1000 and 2000 rejected

Trial	Total Capacity	Time (seconds)	Optimal
1	89.963281	200.871589	93.6519
2	89.743211	199.471652	93.6519
3	89.081309	199.421510	93.6519
4	89.603113	199.821309	93.6519
5	88.998196	198.341215	93.6519

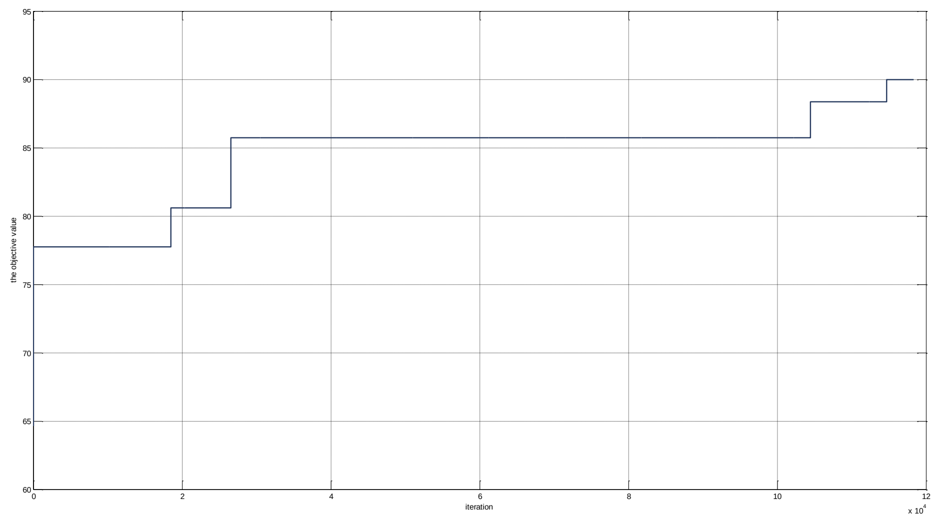

Figure 4-26 Trial 1 - SA Performance (medium problem) - Temp 1000, 2000 rejection

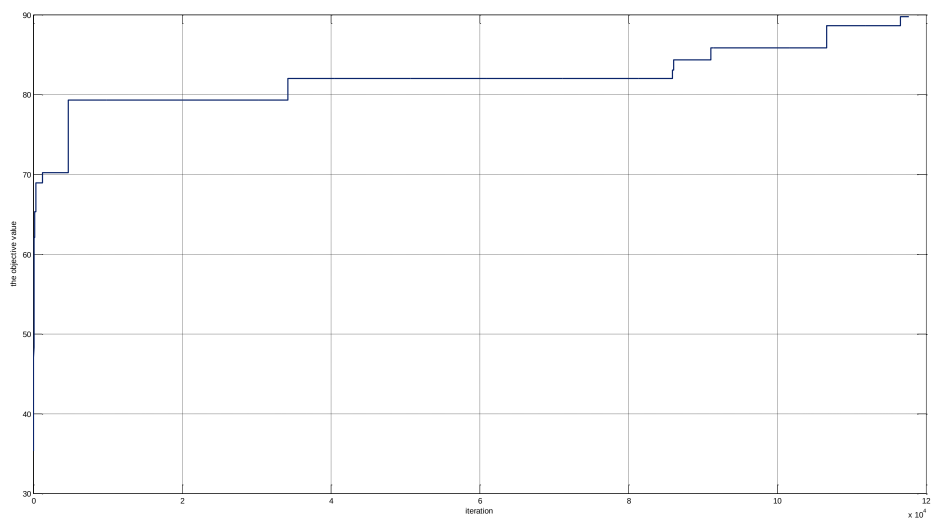

Figure 4-27 Trial 2 - SA Performance (medium problem) - Temp 1000, 2000 rejection

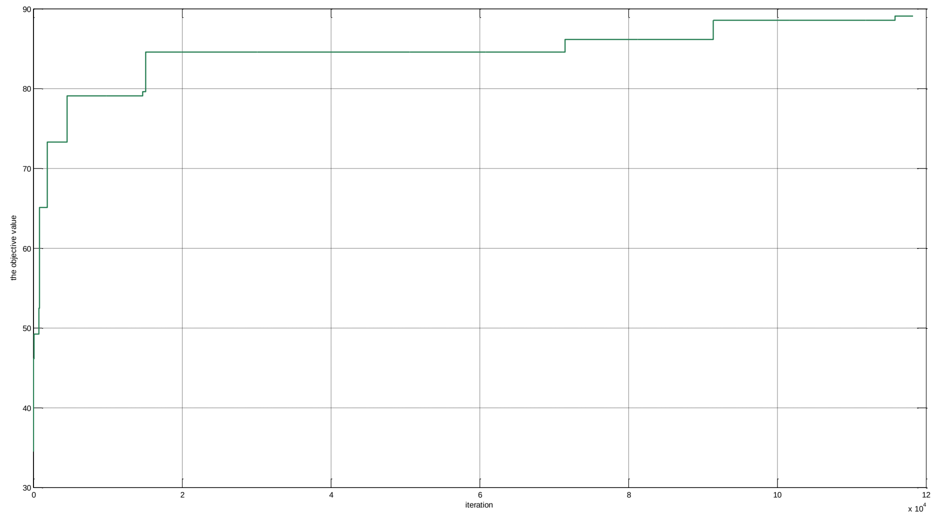

Figure 4-28 Trial 3 - SA Performance (medium problem) - Temp 1000, 2000 rejection

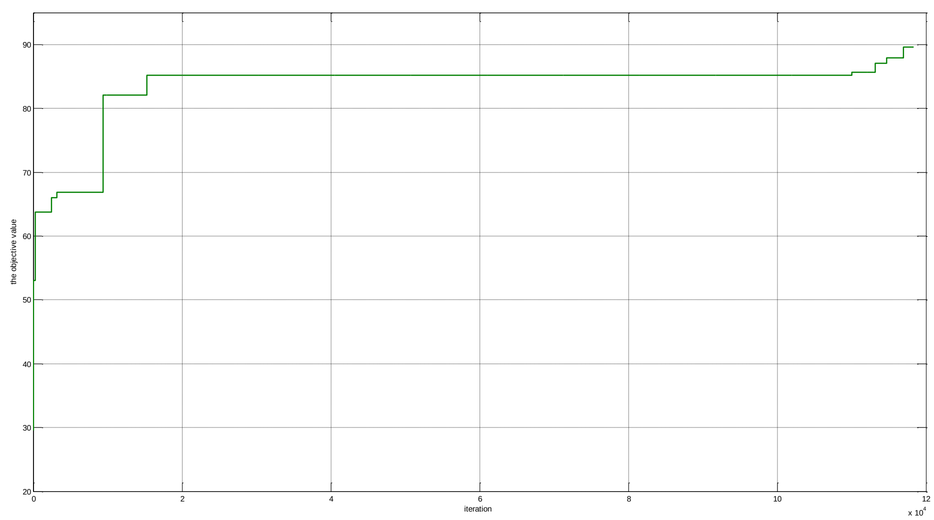

Figure 4-29 Trial 4 - SA Performance (medium problem) - Temp 1000, 2000 rejection

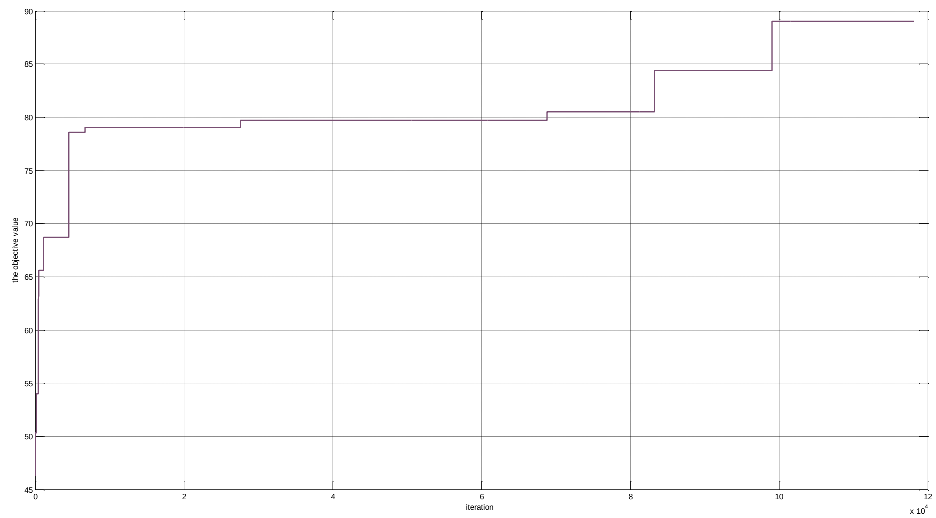

Figure 4-30 Trial 5 - SA Performance (medium problem) - Temp 1000, 2000 rejection

4.6 NUMERICAL EXAMPLE 3 USING SA ALGORITHM (LARGE PROBLEM)

Given H gain matrix (users vs. subcarriers), that has 10 users and 64 subcarriers. The total power (total power budget) is 10. We would like to maximize the total capacity of the OFDMA system such that the fairness constraints are achieved. In this example, we use Simulated Annealing algorithm to solve exactly the same problem 5 times with different initial solutions provided randomly at each different starting temperature.

At starting temperature 500 and the number of rejected points is 1000. We obtain the following results as shown in Table 4-7.

Table 4-7 Results of large size problem at temperature 500 and 1000 rejected

Trial	Total Capacity	Time (seconds)	Optimal
1	207.751307	210.321717	250.4039
2	210.698779	197.052458	250.4039
3	204.435531	197.985441	250.4039
4	206.213637	197.281038	250.4039
5	208.373557	208.972303	250.4039

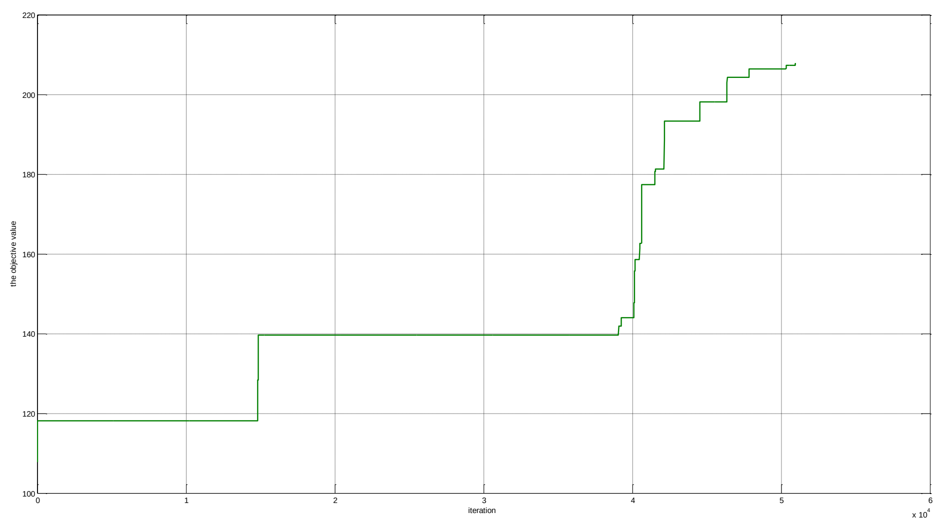

Figure 4-31 Trial 1 - SA Performance (large problem) - Temp 500, 1000 rejection

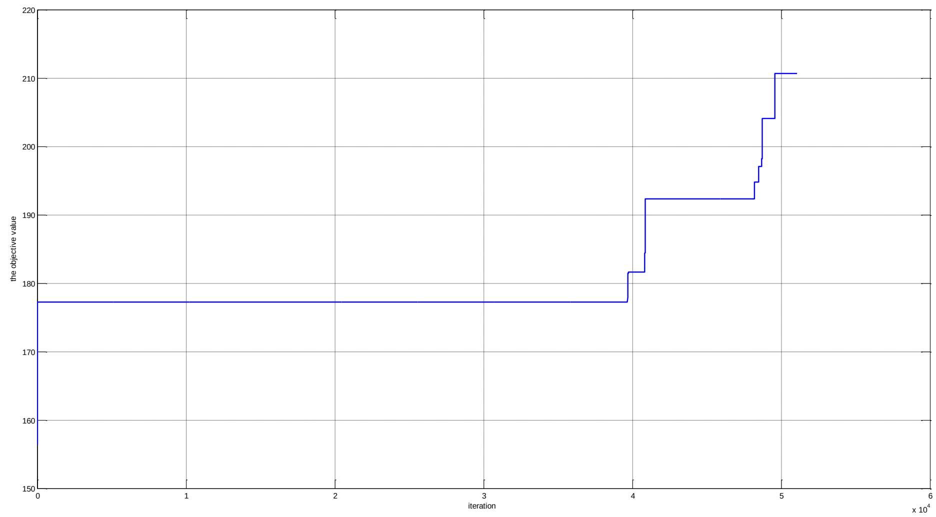

Figure 4-32 Trial 2 - SA Performance (large problem) - Temp 500, 1000 rejection

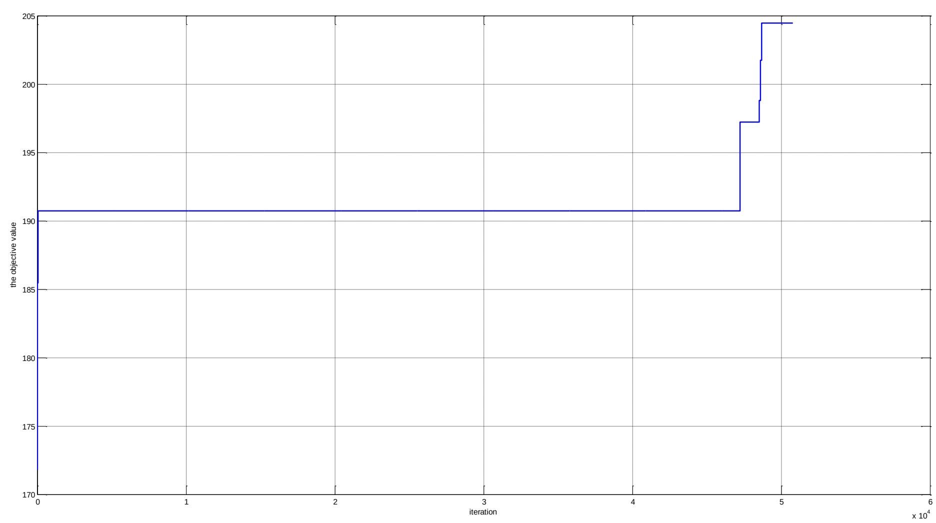

Figure 4-33 Trial 3 - SA Performance (large problem) - Temp 500, 1000 rejection

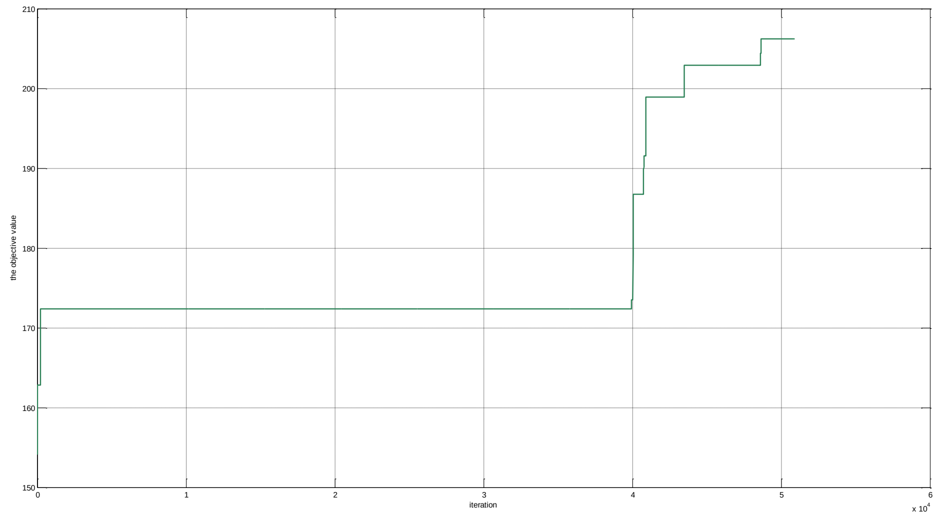

Figure 4-34 Trial 4 - SA Performance (large problem) - Temp 500, 1000 rejection

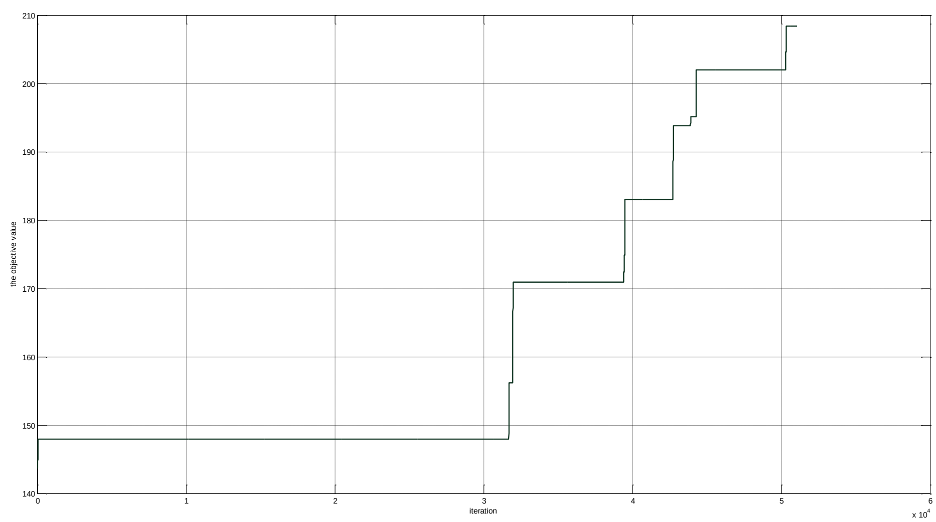

Figure 4-35 Trial 5 - SA Performance (large problem) - Temp 500, 1000 rejection

At starting temperature 750 and the number of rejected points is 1500. We obtain the following results as shown in Table 4-8.

Table 4-8 Results of large size problem at temperature 750 and 1500 rejected

Trial	Total Capacity	Time (seconds)	Optimal
1	207.142154	312.258240	250.4039
2	209.682597	325.242720	250.4039
3	209.884276	311.983464	250.4039
4	214.152465	394.198048	250.4039
5	209.357443	368.356133	250.4039

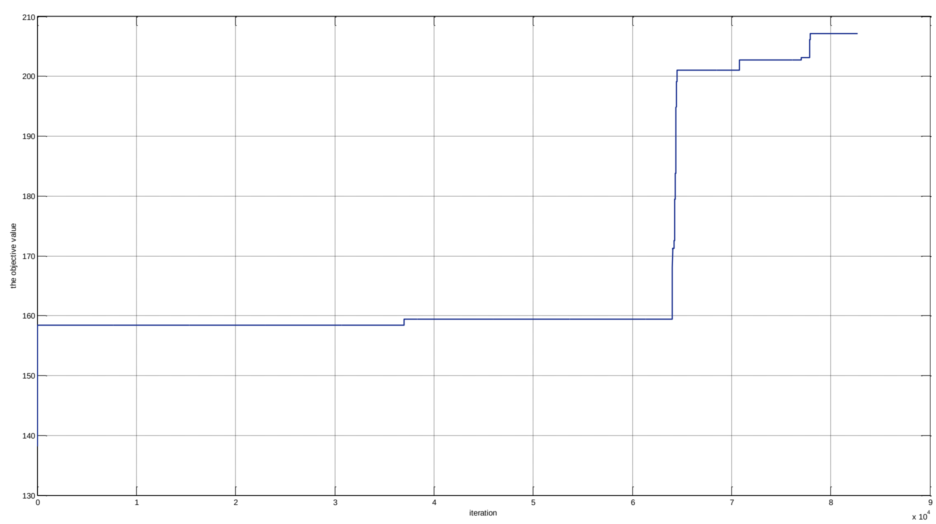

Figure 4-36 Trial 1 - SA Performance (large problem) - Temp 750, 1500 rejection

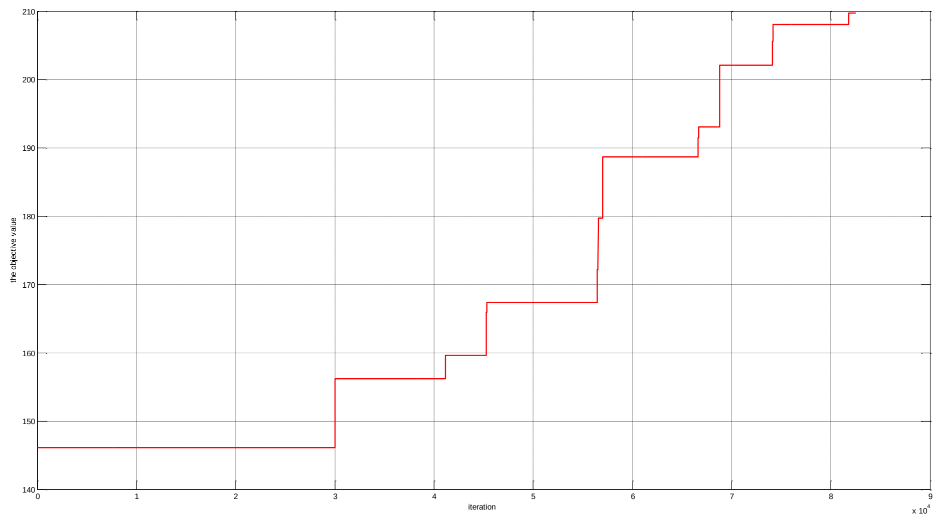

Figure 4-37 Trial 2 - SA Performance (large problem) - Temp 750, 1500 rejection

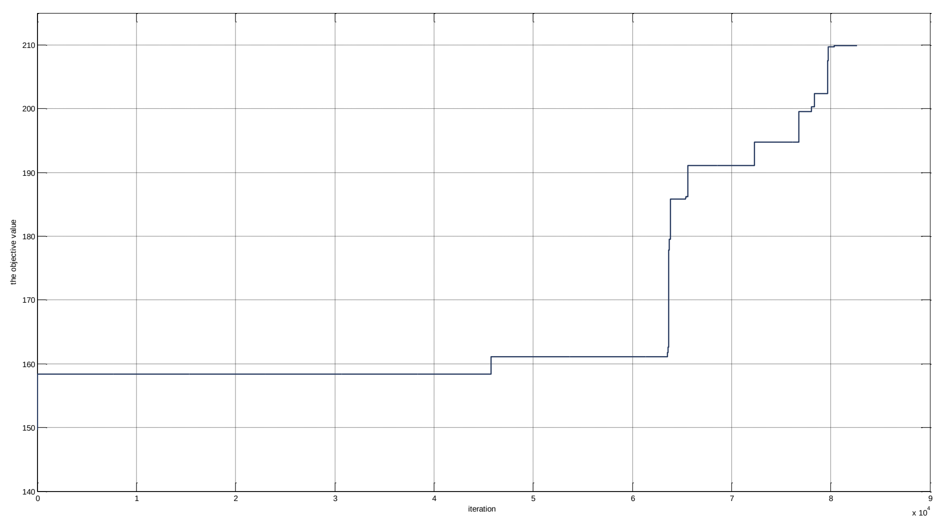

Figure 4-38 Trial 3 - SA Performance (large problem) - Temp 750, 1500 rejection

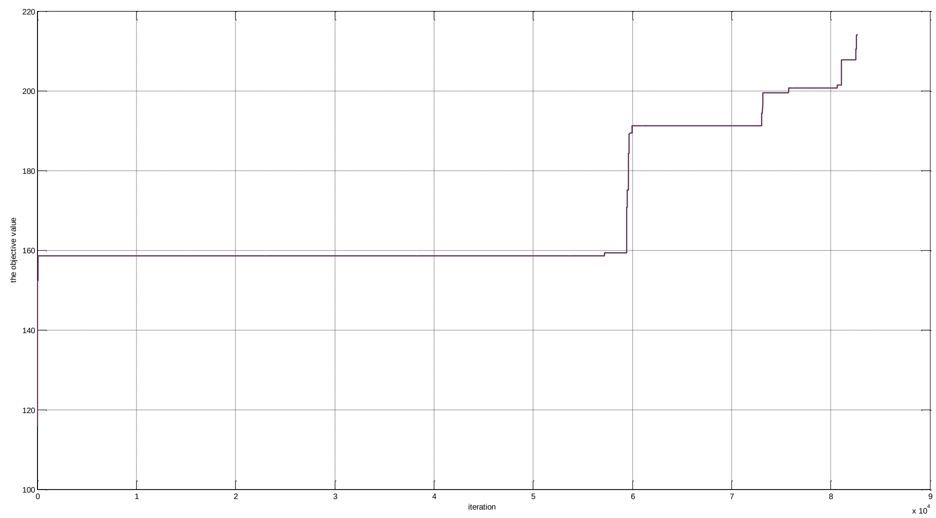

Figure 4-39 Trial 4 - SA Performance (large problem) - Temp 750, 1500 rejection

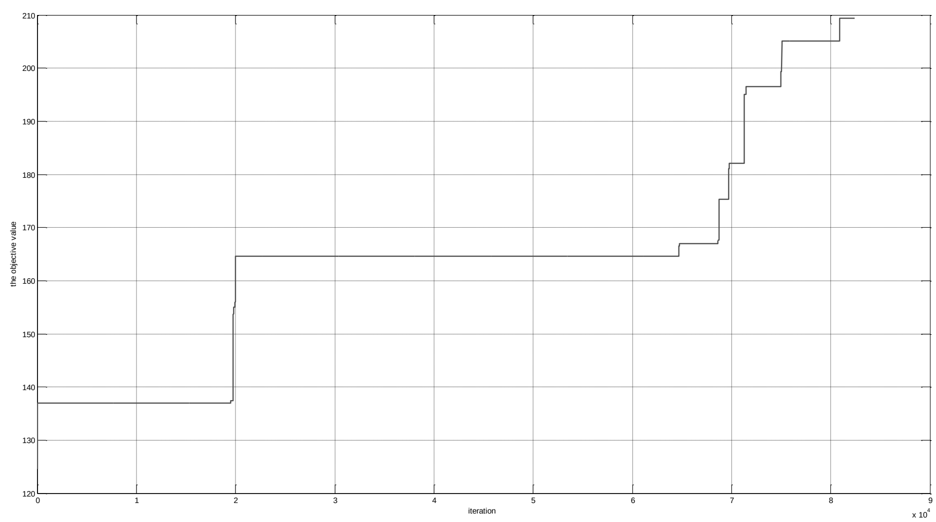

Figure 4-40 Trial 5 - SA Performance (large problem) - Temp 750, 1500 rejection

At starting temperature 1000 and the number of rejected points is 2000. We obtain the following results as shown in Table 4-9.

Table 4-9 Results of large size problem at temperature 1000 and 2000 rejected

Trial	Total Capacity	Time (seconds)	Optimal
1	212.804591	507.059117	250.4039
2	215.936524	517.910236	250.4039
3	208.367986	535.897273	250.4039
4	215.111426	610.238227	250.4039
5	213.895452	432.726179	250.4039

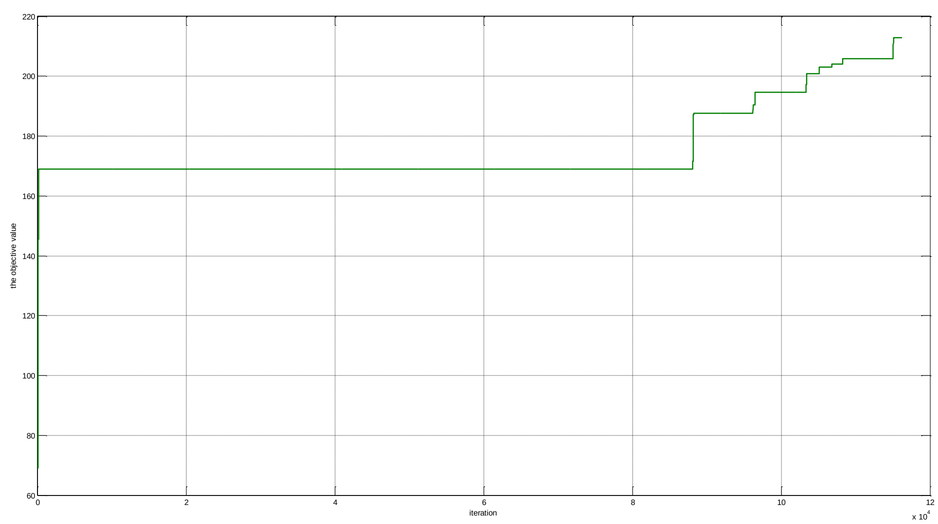

Figure 4-41 Trial 1 - SA Performance (large problem) - Temp 1000, 2000 rejection

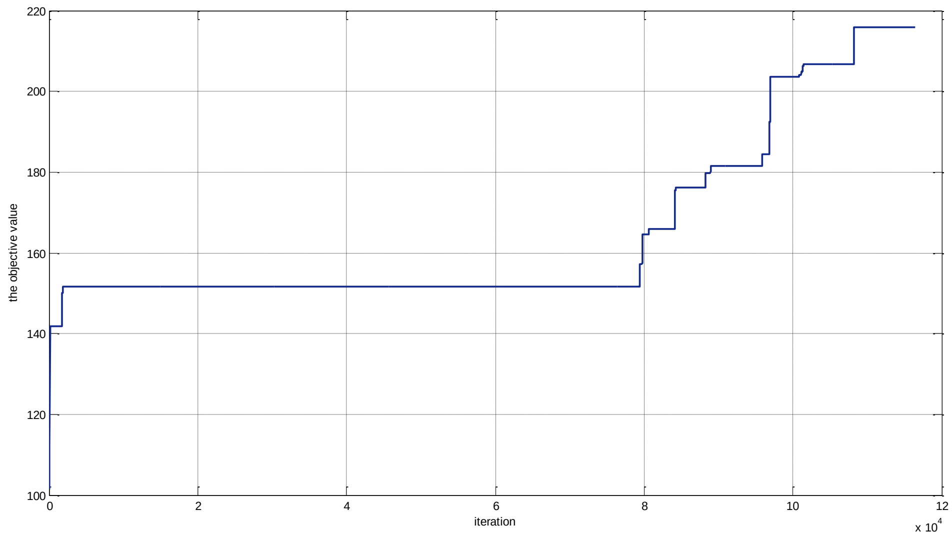

Figure 4-42 Trial 2 - SA Performance (large problem) - Temp 1000, 2000 rejection

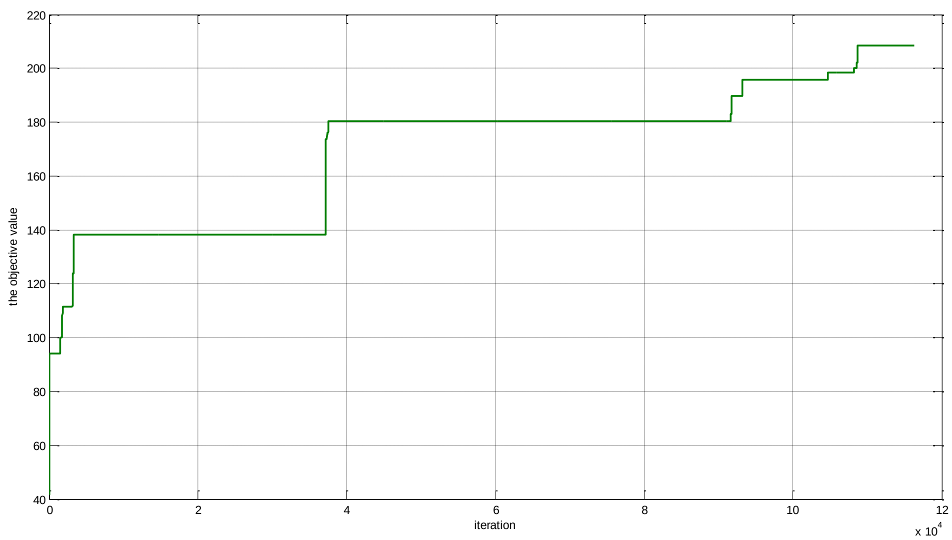

Figure 4-43 Trial 3 - SA Performance (large problem) - Temp 1000, 2000 rejection

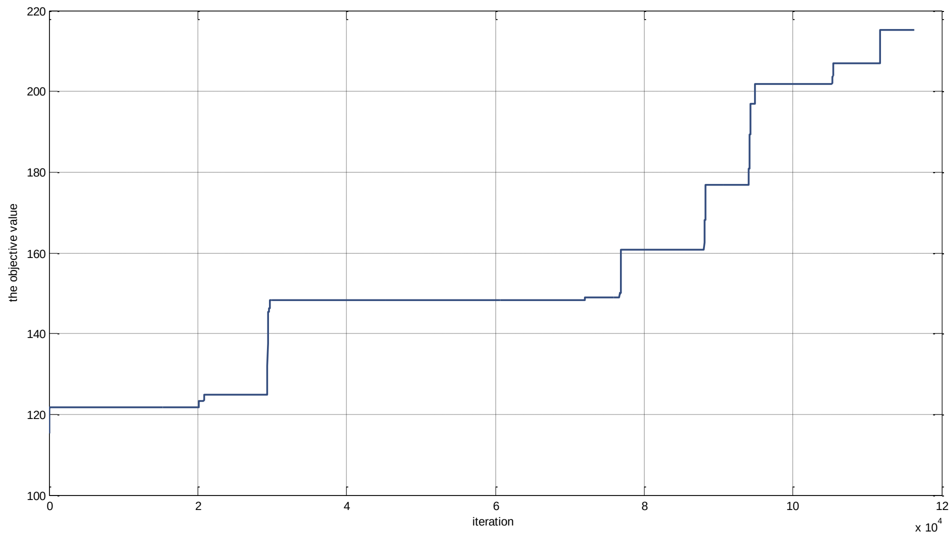

Figure 4-44 Trial 4 - SA Performance (large problem) - Temp 1000, 2000 rejection

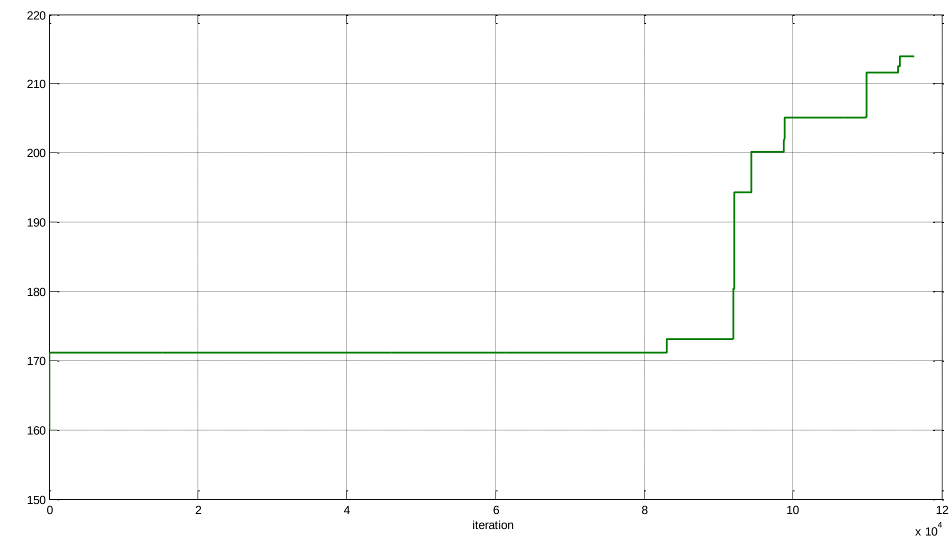

Figure 4-45 Trial 5 - SA Performance (large problem) - Temp 1000, 2000 rejection

4.7 NUMERICAL EXAMPLE 4 USING LINGO 12

Using the same data of numerical example 1 (SA). Indeed, we solve the optimization problem using LINGO 12 as follows:

1. The optimization problem is coded in LINGO programming language to obtain the powers allocation along with the subcarrier allocations. The results are shown in Table(4-11)

Table 4-10 LINGO 12 results for power and subcarrier allocation problem

Problem	(users vs subcarriers)	Total Capacity	Time (seconds)	Optimal
1	(4 vs 8)	51.491	132.24	53.9613
2	(6 vs 16)	86.49326	190.68	93.6519
3	(10 vs 64)	61.51455	240.91	250.4039

We conclude from the results, LINGO12 could not achieve the optimal solution and the quality of the solutions decreased when the number of users and subcarriers are increased.

2. Using the proposed iterative method. The results are shown in Table (4-12).

Table 4-11 LINGO 12 results for power and subcarrier allocation problem (iterative method)

Problem	(users vs subcarriers)	Total Capacity	Optimal
1	(4 vs 8)	51.491	53.9613
2	(6 vs 16)	86.49326	93.6519
3	(10 vs 64)	61.51455	250.4039

The results of an iterative method show no differences with LINGO12 results. Probably, LINGO12 solver is trapped into some local maximum objective because of some standard settings that LINGO12 used for solving optimization problem.

4.8 DISSCUSSION

The Simulated Annealing heuristic search method achieved better performance in general for solving power and subcarrier allocation problem than LINGO12 including an iterative method. The Simulated Annealing heuristic algorithm reached to the optimal solution in small case problem such as 4 users and 8 subcarriers while it gets very close to the optimal solution in bigger size problem, for instance, 10 users and 64 subcarriers. Furthermore, the computational time increases when the starting value of the temperature, number of rejected, and number of users and subcarriers increase.

Chapter 5

CONCLUSION

5.1 CONCLUSION

In this thesis, we started with exploring the properties of the power allocations problem using Karush-Kuhn-Tucker optimality condition by assuming that the allocations of the subcarriers are known in advance. Accordingly, the power allocations problem is reduced to a line search problem with one variable which can be solved in a very short time along with a back substitution method for calculating the needed power for each subcarrier in the system. Moreover, the proposed algorithm guarantees the solution to satisfy the proportional rate constraint with infinite number of subcarriers and users. The fairness conditions may result in an infeasible problem if the total power available for the allocation is not sufficiently large. Therefore, we provide a condition to have a feasible solution to the problem and proposed a new formulation where we relax the fairness constraints using controlled tolerance.

In the second main part of the thesis, we introduce one of the heuristic search methods called Simulated Annealing to solve the subcarrier allocations problem. The results of the Simulated Annealing heuristics algorithm are quite impressive in achieving near optimal solutions. Also, a comparison is made between the results of Simulated Annealing algorithm, LINGO 12 included an iterative method, and the actual optimal solutions.

5.2 FUTURE WORK

For future studies and extensions of the thesis, the researcher can modify the Simulated Annealing heuristics by studying and considering best way to find initial start solution instead of randomly selected.

As a second extension work, the researcher can consider different heuristics methods such as Particle swarms algorithm, ant colony, and bee colony algorithms to solve the power and subcarrier allocations problem.

As a third extension work, the researcher can consider cutting plane method for the subcarrier allocation model.

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